Solving the Traveling Salesman Problem with Demon Algorithms and Variants

by

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Presented at INFORMS Salt Lake City, May 2000
Focus of Paper

- Review several "old" metaheuristics for solving the TSP
  - simulated annealing (SA)
  - threshold accepting (TA)
  - record-to-record travel (RRT)
- Introduce the demon algorithm (DA) and a number of variants
- Perform a computational study
- Present conclusions
Metaheuristics

- Work in this direction began with simulated annealing (1983)

- Metaheuristics for the TSP are often based on the concept of arc exchanges (dating back to the two-opt)

- The starting point was to realize that occasionally accepting arc exchanges that increased tour length was okay
Two-opt Exchange
Simulated Annealing

- Generate an initial tour and set $T$ (temperature)
- Repeat until stopping condition:
  - Generate a new tour and calculate $\Delta E$ (change in energy)
  - If $\Delta E \leq 0$, accept new tour
  - Else, if $\text{rand}(0,1) < \exp(-\Delta E/T)$, accept new tour
  - Else, reject new tour
  - Implement annealing schedule ($T = a \cdot T$)
- The choice of $T$ and $a$ are essential
Deterministic Annealing

- In both TA and RRT, arc exchanges are accepted if $\Delta E$ is less than some bound.

- In TA, we set an initial threshold, $T$, and anneal the threshold as the algorithm progresses ($T = a \times T$)
  - If $\Delta E < T$, accept new solution.

- In RRT, we set the record, $R$, as the length of the best tour found and we set the deviation, $D$
  - If $E < R + D$, accept new solution.
Demon Algorithms

- Wood and Downs developed several demon algorithms for solving the TSP
- In DA, the demon acts as a creditor
- The demon begins with credit $= D > 0$
- Consider an arc exchange
- If $\Delta E < D$, accept new tour and $D = D - \Delta E$
- Arc exchanges with $\Delta E < 0$ build credit
- Arc exchanges with $\Delta E > 0$ reduce credit
Demon Algorithms (continued)

- To encourage minimization, Wood and Downs propose two techniques
  - Impose an upper bound on the demon value, restricting the demon value after energy decreasing moves
  - Anneal the demon value

- Wood and Downs also propose a random component
  - The demon value is a normal random variable centered around the demon mean value
  - All changes in tour length impact the demon mean value
Demon Algorithms (continued)

- This leads to four algorithms (Wood and Downs)
  
  - Bounded demon algorithm (BD)
  - Randomized bounded demon algorithm (RBD)
  - Annealed demon algorithm (AD)
  - Randomized annealed demon algorithm (RAD)
New Demon Algorithms

- Two new techniques come to mind (Pepper et al.)
  - Annealed bounded demon algorithm (ABD)
  - Randomized annealed bounded demon algorithm (RABD)

- The idea is to impose a bound on the demon value (or demon mean value) and anneal that bound in ABD and RABD

- For RAD and RABD, anneal both the bound on the demon mean and the standard deviation. This leads to two additional algorithms, ADH and ABDH
Computational Study

- Eleven algorithms in all
- We selected 29 instances from TSPLIB
- The instances range in size from 105 to 1,432 nodes
- The instances have different structures
- Each algorithm was applied 25 times to each instance from a randomized greedy start
- Best and average performance and running time statistics were gathered
Parameter Settings

- We selected three test instances
  - Stage 1 GA determines a set of parameter values (parameter vector) for each test instance
  - Stage 2 GA combines the parameter vectors, minimizing tour lengths across all test instances simultaneously

- Resulting parameter vector is applied to all 29 instances
## Final Values of Parameters

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$if$</th>
<th>$sdf$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
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<tr>
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<td>0.9646</td>
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<tr>
<td>TA</td>
<td>0.0722</td>
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<td>0.9515</td>
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<tr>
<td>RRT</td>
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<td></td>
<td></td>
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<tr>
<td>BD</td>
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<tr>
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# Results

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>NBS</th>
<th>Avg. Perf.</th>
<th>ET</th>
</tr>
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<td>36.6</td>
</tr>
</tbody>
</table>

NBS = # of instances where algorithm generates the best tour  
Avg. Performance = average % above optimal solution over 29 instances  
ET = total execution time in hours
Conclusions

- First extensive computational study of demon algorithms and variants
- The computational results are mixed
  - ABD and RABD are competitive with SA
  - The other variants are not
- Future work: Do smarter DA variants exist?

Note:  ABD = annealed bounded demon algorithm
       RABD = randomized annealed bounded demon algorithm