Algorithms, Logistics, and the New Economy

by

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Distinguished Scholar-Teacher Lecture
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Focus

- Discuss algorithms and computational effort
- Contrast optimal algorithms with heuristics
- Illustrate several heuristic strategies
- Apply ideas to operational problems in logistics management
- Explore some connections to the New Economy
  - MapQuest
  - RouteSmart
- Conclusions
The Transportation Problem

- We begin with a simple problem from logistics

- There are \( m \) supply points with items available to be shipped to \( n \) demand points

- Plant \( i \) can ship at most \( S_i \) items, and warehouse \( j \) requires at least \( D_j \) items

- The cost of shipping each unit from plant \( i \) to demand point \( j \) is provided

- The objective is to select a routing plan that minimizes total transportation costs
A Small Transportation Problem

Supply | Plant | Warehouse | Demand
-------|-------|-----------|-------
300    | A     | X         | 600   
600    | B     | Y         | 300   
500    | C     | Z         | 500   

flow from plants to warehouses
# A Tabular Representation

<table>
<thead>
<tr>
<th>Plant</th>
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| Demand | 600 | 300 | 500 | 1400 |

Warehouse
Ad-Hoc Solution Procedure

1. Avoid the most costly route
2. Fully utilize the cheapest route

Warehouse

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Ad-Hoc Solution Procedure -- continued

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- Total cost = 300 x 0 + 300 x 1 + 300 x 6 + 500 x 6 = $5100
- This approach is reasonable, but a much better solution exists
Optimal Solution

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- Total cost = 300 x 1 + 400 x 1 + 200 x 3 + 200 x 3 + 300 x 7 = $4000
- Note 1. Nothing is sent along the zero cost route
- Note 2. Most expensive route is used as much as possible
- Note 3. We can obtain this solution using linear programming
Observations

- We examined a small, toy problem
- We applied common sense and intuition in our ad hoc solution procedure
- The resulting cost was 27.5% above the cheapest (optimal) cost
- Common sense and intuition were not good enough
- In practice, transportation problems are much larger and need to be solved repeatedly
- If we can’t use linear programming, we should apply a systematic procedure (a heuristic) that consistently generates feasible, near-optimal solutions
- The minimum-matrix method is one such heuristic
A Small Diversion

- Linear programming (LP) is one of the fundamental tools of management science
- George Dantzig developed the simplex method in 1947
- Dantzig was born in 1914
- His father, Tobias Dantzig, taught in our Math Dept. for many years
- George wrote his dissertation in mathematical statistics at Berkeley
- He worked in the Pentagon during WWII
- He spent many years as a Professor at Stanford
- He received an Honorary Doctorate from us in 1976
- About the 1975 Nobel Prize in Economics
The Minimum-Matrix Method

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- Find the cheapest route
- Send as much as possible along this route
The Minimum-Matrix Method -- continued

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Total cost = 300 x 0 + 300 x 1 + 300 x 3 + 300 x 7 + 200 x 6 = $4500

The resulting cost is only 12.5% above the optimal cost
Logistics Management

- Logistic management involves the cost-effective flow and storage of goods, services, and personnel from origin to destination according to a set of pre-specified requirements.

- The transportation problem is a simple example of logistics management.

- The maximum flow, shortest path, and traveling salesman problem are others.
Maximum Flow in a Railway Network

- A large number of commuters travel from a to g every day
- Three routes are shown below

- Each section of the track (each link) has a capacity (number of trains per hour) based on safety considerations
- Suppose each capacity is 10
- How many trains per hour can reach g?
- How can the number be increased by changing the capacity of a single link?
Find the Most Economical Package - Delivery Route from San Francisco to New York
The Traveling Salesman Problem

- Imagine a suburban college campus with 140 separate buildings scattered over 800 acres of land
- To promote safety, an experienced security guard must inspect each building every evening
- The goal is to sequence the 140 buildings so that the total time (travel time plus inspection time) is minimized
- This is an example of the well-known TSP

Original problem

Possible solution
Analysis of Algorithms

Definitions

- Algorithm – method for solving a class of problems on a computer
- Optimal algorithm – verifiable optimal solution
- Heuristic algorithm – feasible solution

Performance Measures

- Number of basic computations / Running time
- Computational effort
  --- Problem size
  --- Player one
  --- Player two
Computational Effort as a Function of Problem Size

- $2^n$
- $n^3$
- $n^2$
- $n\log_2 n$
- $n$

Problem size

Computational effort
Good vs. Bad Algorithms

- **Terminology**
  - Researchers have emphasized the importance of finding polynomial time algorithms, by referring to all such polynomial algorithms as inherently good.
  - Algorithms that are *not* polynomially bounded, are labeled inherently bad.

- **Good Optimal Algorithms Exist for these Problems**
  - Transportation problem
  - Maximum flow problem
  - Shortest path problem
  - Linear programming
Good vs. Bad Algorithms -- continued

- **Good Optimal Algorithms Don’t Exist for these Problems**
  - Traveling salesman problem (TSP)
  - Complex logistics problems (e.g., vehicle routing, vehicle fleet management, delivery with time-windows, pickup and delivery systems, integration of inventory and transportation)

- **Why Focus on Heuristic Algorithms?**
  - For the above problems, optimal algorithms are not practical
  - Efficient, near-optimal heuristics are needed to solve real-world problems
  - The key is to find fast, high-quality heuristic algorithms
Some Heuristic Algorithm Strategies

- Aggregate / disaggregate
- Divide and conquer
- Allow uphill moves for minimization problems
The Elastic Net Algorithm
[Durbin and Willshaw 87]

- Find the shortest TSP tour through n points

- Algorithm
  - Introduce a rubber band as a small circle around the center of gravity of the points
  - Stretch the rubber band towards the points by minimizing an energy function of the form:
    \[ E = -\alpha K \sum_{i=1}^{n} \ln \sum_{j=1}^{m} e^{-||x_i - y_j||^2 / 2K^2} + \beta \sum_{j=1}^{m} ||y_j - y_{j+1}|| \]
    where \( K \to 0 \) and \( ||a - b|| = (a_1 - b_1)^2 + (a_2 - b_2)^2 \)
  - The rubber band is stretched over time, using ideas from calculus, until the rubber band and the points coincide
Key Observations

- Explain the two terms of the energy function

- Procedure is iterative

- At each iteration, the most burdensome step involves $n \times m$ exponentiations

- To speed up the algorithm, we must address this bottleneck
The Concept of Rubber Band Points
Aggregation / Disaggregation Strategy

- Reduce both n and m by aggregating the points
- Slowly disaggregate
- Define center of gravity
## Where Does the Speedup Come From?

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Original Elastic Net</th>
<th>Accelerated Elastic Net</th>
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<tbody>
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Practical Observations

- The approximate speedup for 500 points is 16.4

- The approximate speedup for 1000 points is 17.2

- Impact: 10 minutes vs. nearly 3 hours

- The accelerated elastic net is faster and yields higher quality tours
Divide and Conquer

a. Horizontal bisection

b. Subproblems solved

c. Patched solution

d. Recursive divide and conquer
Allow Uphill Moves
Geographic Information Systems

- A GIS is a computer-based tool for the manipulation and spatial analysis of geographic data
- The field is about 30 years old
- Annual sales of GIS software exceed $1 billion
- The growth rate in sales has been approx. 25% per year
- Three GIS-based products of interest: MapQuest, Microsoft MapPoint 2001, and RouteSmart
RouteSmart Technologies, Inc.

- Company designs and sells vehicle routing and scheduling software
- Small Md. Firm, based in Columbia, founded in 1980
- Owned by a large NY civil engineering company
- I’ve been associated with the firm from the beginning
- Currently run by Larry Levy -- my newspaper boy in 1978 & 1979
- RouteSmart clients include the NY Times, The WSJ, and the San Francisco Chronicle
- RouteSmart has major installations in the newspaper, utility, sanitation, and small package delivery industries
- Larry Levy graduated (B.S. & Ph.D.) from our Business School in the 1980s
The Size of the Small Package Delivery Industry

- **FedEx**
  - 5 million shipments / business day
  - 55,000 vehicles in the ground fleet

- **UPS**
  - 13.1 million packages daily
  - 150,000 vehicles in the ground fleet

- **USPS**
  - More than 200 billion pieces of mail over 302 delivery days
  - 237,000 routes, 6 days a week
Business-to-Consumer Ecommerce

- By 2004, home delivery revenues from B2C ecommerce alone are expected to exceed $125 billion
- Competition among FedEx, UPS, and the USPS is intense
- With express mail, air transportation is key
- FedEx took the early lead and UPS had to play catch-up
- With B2C ecommerce, ground transportation is key
- UPS has the advantage in fleet size and FedEx must play catch-up
FedEx Home Delivery and RouteSmart

- FedEx Ground (formerly the RPS Group) focuses on B2B
- Opened Home Delivery in March 2000
- 68 terminals in U.S. currently
- 150 additional terminals to open in 2001
- 20,000 packages / business day (growing steadily)
- Focus on responding to B2C ecommerce
- All packages are routed for delivery using RouteSmart
Streets Pre-assigned to Routes
Sequenced Stops as Crow Flies (Streets Suppressed)
Sequenced Stops over the Street Network (Streets Suppressed)
Detailed Display of a Single Travel Path (Gray Streets)
Conclusions

- Logistics and distribution management has often been taken for granted.
- This has changed in the last decade or so.
- We observed that even “simple” logistics problems can be difficult to solve.
- Common sense and intuition are not good enough.
- Systematic procedures (e.g., heuristics) often need to be designed and applied.
- We discussed several strategies for designing heuristics for solving the TSP.
- These strategies can be used to tackle important real-world problems (e.g., the FedEx Home Delivery problem).
- A wide variety of more complex logistics and distribution management problems can be approached in the same way.
Recommended Reading


Epilogue: The 1975 Nobel Prize in Economics

- It was given for work in LP, but not to Dantzig
- Instead, it was awarded to Koopmans (Yale) and Kantorovich (USSR)
- Prize was $240K – to be split
- Many were outraged at Dantzig’s omission
- Koopmans considered declining the Prize
- In the end, he accepted the Prize and immediately donated $40K to a Think Tank in Austria with which Dantzig was affiliated
- He was left with $80K (one-third of $240K) which was the most he felt he deserved