Introduction

- Open Vehicle Routing Problem (OVRP)
  
  A vehicle does not return to the depot after servicing the last customer on a route

  Each route in the OVRP is a Hamiltonian path

- Two objectives
  
  Minimize the total number of vehicles
  
  Minimize the total distance traveled

  We believe the cost of an extra vehicle far exceeds the reduction in distance that can be achieved by an additional route
Introduction

Real-world applications of the OVRP

FedEx generates incomplete delivery routes for airplanes (Bodin et al. 1983)

FedEx Home Delivery service to residential-only customers (Levy 2005)

Newspaper home delivery problem (Levy 2005)

If a company contracts drivers with vehicles and drivers are not required or paid to return to the depot, then the application fits the OVRP framework
Algorithms for the OVRP

Since 2000, seven algorithms have been developed to solve OVRP

Two use threshold accepting
Three use tabu search
One uses large neighborhood search
One uses the minimum spanning tree
Literature Review

Seven algorithms for the OVRP

Sariklis and Powell (2000)
   Cluster First, Route Second (CFRS)

Brandao (2004)
   Tabu Search Algorithm (TSA)

   Adaptive Memory-based Tabu Search BoneRoute (BR)

   Backtracking Adaptive Threshold Accepting (BATA)
Literature Review

▶ Seven algorithms for the OVRP

Tarantilis, Ioannou, Kiranoudis, and Prastacos (2005)
   List-Based Threshold Accepting (LBTA)

Fu, Eglese, and Li (2005)
   Tabu Search Heuristic (TS)

Pisinger and Ropke (2005)
   Adaptive Large Neighborhood Search (ALNS)
Record-to-Record Travel

► A deterministic variant of simulated annealing developed by Dueck (1993)

► Framework of RTR (for a minimization problem)
  
  Record (R) : Best solution found so far
  Deviation (D) : Amount of uphill move allowed
      \( D = k\% \times R \)
  
  Rule : If \( \text{Obj} (S') < R + D \), then solution \( S \) is replaced by \( S' \)
Solving the OVRP with RTR Travel

- Adapted from RTR for solving large-scale VRPs (Li, Golden, and Wasil 2005) to solve the open vehicle routing problem (ORTR)

- Features of ORTR
  - Fixed-length neighbor list with 20 customers
    (tradeoff between running time and solution quality)
  - Sweep algorithm to generate an initial solution
    with a minimum number of vehicles
  - Combine two routes (if possible) to reduce the total
    number of vehicles even if the total distance increases
Computational Results

- Benchmark data sets
  - 16 test problems
    - C1 to C14 from Christofides et al. (1979)
    - F11, F12 from Fisher (1994)
  - 50 to 199 customers
  - 7 problems have a route-length restriction

- ORTR coded in Java
  - Athlon 1 GHz, 256 MB RAM, Linux
## Illustrative results

<table>
<thead>
<tr>
<th>Problem</th>
<th>$K_{min}$</th>
<th>Minimize Vehicles with Least Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>5</td>
<td>408.5 TSF</td>
</tr>
<tr>
<td>C2</td>
<td>10</td>
<td>567.14 ALNS 25K, ALNS 50K, ORTR</td>
</tr>
<tr>
<td>C6</td>
<td>5</td>
<td>400.6 TSF (6 vehicles)</td>
</tr>
<tr>
<td>C14</td>
<td>10</td>
<td>591.87 ALNS 25K, ALNS 50K, ORTR (11 vehicles)</td>
</tr>
<tr>
<td>F12</td>
<td>7</td>
<td>769.66 ORTR</td>
</tr>
</tbody>
</table>

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<tr>
<td>C6</td>
<td>5</td>
<td>400.6 TSF (6 vehicles)</td>
</tr>
<tr>
<td>C14</td>
<td>10</td>
<td>469.3 TSR (11 vehicles)</td>
</tr>
<tr>
<td>F12</td>
<td>7</td>
<td>769.66 ORTR</td>
</tr>
</tbody>
</table>
## Computational Results

### Aggregate results for top four procedures

<table>
<thead>
<tr>
<th></th>
<th>TSR</th>
<th>ALNS 25K</th>
<th>ALNS 50K</th>
<th>ORTR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Vehicles (sum of $K_{min}$ = 147)</td>
<td>162</td>
<td>156</td>
<td>156</td>
<td>159</td>
</tr>
<tr>
<td>Total Distance</td>
<td>10,123</td>
<td>10,199</td>
<td>10,194</td>
<td>10,191</td>
</tr>
<tr>
<td>Time (s)</td>
<td>6,347</td>
<td>13,350</td>
<td>22,200</td>
<td>1,756</td>
</tr>
</tbody>
</table>
Computational Results

► When the number of vehicles is minimized
   ALNS 50K generated best solution to 9 problems (56%)
   ALNS 25K 7 problems (44%)
   ORTR 5 problems (31%)

► When the total distance traveled is minimized
   TSR generated best solution to 5 problems (31%)
   ORTR 4 problems (25%)
Computational Results

▶ ORTR solutions

Problem C2, n = 75, solution value = 567.14
Computational Results

► ORTR solutions

Problem C14, n = 100, solution value = 591.87
Computational Results

▸ ORTR solutions

Problem F12, n = 134, solution value = 769.66
Large-Scale Test Problems

- New test problems
  - 8 problems
    - LSVRPs from Golden et al. (1998)
  - 200 to 480 customers
  - No route-length restriction
  - Geometric symmetry
    - Customers in concentric circles around the depot
  - Visually estimate solutions
Large-Scale Test Problems

- Visually estimated solutions

Problem O1, $n = 200$, estimated solution value = 6151.77
Large-Scale Test Problems

- Visually estimated solutions
  Problem O8, n = 480, estimated solution value = 12513.11
## Computational Results

### Results for ORTR on new test problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>n</th>
<th>C</th>
<th>Kmin</th>
<th>ORTR</th>
<th>Time(s)</th>
<th>% Improvement over Estimated Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>200</td>
<td>900</td>
<td>5</td>
<td>6018.52</td>
<td>365.3</td>
<td>2.17</td>
</tr>
<tr>
<td>O2</td>
<td>240</td>
<td>550</td>
<td>9</td>
<td>4584.55</td>
<td>439.6</td>
<td>4.20</td>
</tr>
<tr>
<td>O3</td>
<td>280</td>
<td>900</td>
<td>7</td>
<td>7732.85</td>
<td>492.8</td>
<td>1.28</td>
</tr>
<tr>
<td>O4</td>
<td>320</td>
<td>700</td>
<td>10</td>
<td>7291.89</td>
<td>573.6</td>
<td>0.64</td>
</tr>
<tr>
<td>O5</td>
<td>360</td>
<td>900</td>
<td>8</td>
<td>9197.61</td>
<td>766.5</td>
<td>1.14</td>
</tr>
<tr>
<td>O6</td>
<td>400</td>
<td>900</td>
<td>9</td>
<td>9803.80</td>
<td>977.2</td>
<td>1.22</td>
</tr>
<tr>
<td>O7</td>
<td>440</td>
<td>900</td>
<td>10</td>
<td>10374.97</td>
<td>935.4</td>
<td>1.26</td>
</tr>
<tr>
<td>O8</td>
<td>480</td>
<td>1000</td>
<td>10</td>
<td>12429.56</td>
<td>1126.8</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Total Vehicles 68  Estimated solutions used 72

Average 1.57

C is vehicle capacity
Large-Scale Test Problems

- ORTR solutions

Problem O1, n = 200, estimated solution value = 6018.52
Large-Scale Test Problems

- ORTR solutions

Problem O1, $n = 480$, estimated solution value = $12429.56$
Conclusions

- Increased interest in the OVRP in last five years
  Contractors used to deliver packages and newspapers
  Wide variety of new algorithms to solve problems

- Three algorithms were accurate
  Adaptive large neighborhood search (ALNS 25K, ALNS 50K)
  Tabu search (TSR)
  Record-to-record travel (ORTR)

- Generated eight large-scale test problems
  ORTR found good solutions in a few minutes