Interesting Research in Vehicle Routing and Healthcare Analytics

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Outline of Talk

- Some personal remarks
- Vehicle Routing
  - The Hierarchical Traveling Salesman Problem (HTSP)
- Healthcare Analytics
  - The Effects of Bed Utilization on Discharge and Readmission Rates
- Conclusions
Grand Hotel
Convento di Amalfi
Introduction to the HTSP

- Consider the distribution of relief aid
  - E.g., food, bottled water, blankets, or medical packs
- The goal is to satisfy demand for relief supplies at many locations
  - Try to minimize cost
  - Take the urgency of each location into account
A Simple Model for Humanitarian Relief Routing

- Suppose we have a single vehicle which has enough capacity to satisfy the needs at all demand locations from a single depot.
- Each node (location) has a known demand (for a single product called an aid package) and a known priority.
  - Priority indicates urgency.
  - Typically, nodes with higher priorities need to be visited before lower priority nodes.
Node Priorities

- Priority 1 nodes are in most urgent need of service
- To begin, we assume
  - Priority 1 nodes must be served before priority 2 nodes
  - Priority 2 nodes must be served before priority 3 nodes, and so on
  - Visits to nodes must strictly obey the node priorities
The Hierarchical Traveling Salesman Problem

- We call this model the Hierarchical Traveling Salesman Problem (HTSP)
- Despite the model’s simplicity, it allows us to explore the fundamental tradeoff between efficiency (distance) and priority (or urgency) in humanitarian relief and related routing problems
- A key result emerges from comparing the HTSP and TSP in terms of worst-case behavior
Four Scenarios for Node Priorities

(a) Tsunami
(b) Earthquake
(c) Hurricane
(d) Flood

LEGEND: D Depot  Node with priority p
Literature Review

- Psaraftis (1980): precedence constrained TSP
- Fiala Tomlin, Pulleyblank (1992): precedence constrained helicopter routing
- Guttman-Beck et al. (2000): clustered traveling salesman problem
- Campbell et al. (2008): relief routing
- Balcik et al. (2008): last mile distribution
A Relaxed Version of the HTSP

- Definition: The d-relaxed priority rule adds operational flexibility by allowing the vehicle to visit nodes of priority \( \pi + 1, \ldots, \pi + d \) (if these priorities exist in the given instance) but not priority \( \pi + d + \ell \) for \( \ell \geq 1 \) before visiting all nodes of priority \( \pi \) (for \( \pi = 1, 2, \ldots, P \))
- When \( d=0 \), we have the strict HTSP
- When \( d=P-1 \), we have the TSP (i.e., we can ignore node priorities)
Efficiency vs. Priority

HTSP\( (d=3) \): Optimal Tour Length = 3.56

HTSP\( (d=1) \): Optimal Tour Length = 5.29
Main Results

- Let $P$ be the number of priority classes
- Assume the triangle inequality holds
- Let $Z_{d,P}^*$ and $Z_{TSP}^*$ be the optimal tour length (distance) for the HTSP with the d-relaxed priority rule and for the TSP (without priorities), respectively
- We obtain the following results
  
  \[(a) Z_{0,P}^* \leq PZ_{TSP}^* \]
  \[(b) Z_{d,P}^* \leq \left\lfloor \frac{P}{d+1} \right\rfloor Z_{TSP}^* \]
Sketch of Proof (a)

Tour $\tau^*$
Length $= Z^*_{\text{TSP}}$
Sketch of Proof (a)

- Construct tours $\tau(1)$, $\tau(2)$, and $\tau(3)$
- Visit nodes in the same order as they appear in $\tau^*$
- From the triangle inequality, the lengths of $\tau(1)$, $\tau(2)$, and $\tau(3)$ are each $\leq Z^*_{\text{TSP}}$
- It is easy to construct a feasible solution $\tau$ to the HTSP from $\tau(1)$, $\tau(2)$, and $\tau(3)$
- The length of $\tau \leq \sum_{i=1}^{3} \{\text{length of } \tau(i)\} \leq 3Z^*_{\text{TSP}}$
Sketch of Proof of (b)

Tour $\tau^*$
Length $= Z^*_\text{TSP}$
Sketch of Proof of (b)

- In our example, $P=4$ and $d=1$
- In the worst case, we can’t visit a priority 3 node until we have visited all priority 1 and 2 nodes
- Visit nodes in the same order as they appear in $\tau^*$
- $\tau(1)$ includes priority 1 and 2 nodes
- $\tau(2)$ includes priority 3 and 4 nodes
- As before, we can construct $\tau$ from $\tau(1)$ and $\tau(2)$
- The length of $\tau \leq \sum_{i=1}^{2} \{\text{length of } \tau(i)\} \leq 2Z^*_{\text{TSP}}$
The General Result and Two Special Cases

- \( Z^*_{d,P} \leq \left\lfloor \frac{P}{d+1} \right\rfloor Z^*_{TSP} \)

- If \( d=0 \), we have part (a)

- If \( d=P-1 \), then \( Z^*_{d,P} = Z^*_{TSP} \)
Worst-case Example

[Diagram showing a network with nodes and edges labeled with values.]
Several Observations

- Observation 1. The worst-case example shows that the bounds in (a) and (b) are tight and cannot be improved.

- Observation 2. We can “solve” a TSP over the entire set of nodes using our favorite TSP heuristic and obtain a feasible tour for the HTSP by following the part (b) proof.

- Observation 3. Suppose we select Christofides’ heuristic and let $Z_{d,P}^h$ be the length of the resulting feasible solution to the HTSP, then we have $Z_{d,P}^h \leq \frac{3}{2} \cdot \left\lceil \frac{P}{d+1} \right\rceil Z_{TSP}^*$.
Observations and Extensions

- Observation 4. The HTSP (with $d=0$) can be modeled and solved as an ATSP

- Observation 5. Other applications of the HTSP include routing of service technicians and routing of unmanned aerial vehicles

- We can obtain similar worst-case results (with tight bounds) for the HTSP on the line and the Hierarchical Chinese Postman Problem (HCPP)
Extensions and Future Work

- The HTSP and several generalizations have been formulated as mixed integer programs.

- HTSP instances with 30 or so nodes were solved to optimality using CPLEX.

- Future work:
  - The Hierarchical Vehicle Routing Problem (HVRP)
  - A multi-day planning horizon
  - Uncertainty with respect to node priorities
Emergence of Healthcare Analytics within INFORMS

Number of Healthcare Talks at INFORMS Annual Meetings

Above numbers courtesy of Brian Denton
Strength in Numbers

- There is more healthcare data available than ever before
  - Careful analysis of healthcare data can lead to smarter decisions, better quality healthcare, and cost savings

- A larger number of healthcare decision makers have MBAs than ever before
  - They understand that we can help

- A larger number of us in OR/OM are working on healthcare applications than ever before
The Effects of Bed Utilization on Discharge and Readmission Rates

- Many hospital resources are required for surgery
  - Operating rooms
  - Nurses & Physicians
  - Anesthesia team
  - Post-operative beds for recovery

- If downstream beds are unavailable, surgery might be postponed or cancelled

- Surgeons decide when patients are discharged
  - Surgeons are paid to do surgery
Research Question 1

- Does the utilization of downstream beds affect the discharge decisions of surgeons?
  - Hypothesis: There is an increased discharge rate on days when post-operative utilization is high.
Data

- Data collected on every surgery performed at a large US hospital from Jan 1, 2007 to May 31, 2007
- 7808 patients, of which 6470 were admitted to the hospital and stayed for at least one night
- These patients stayed a total of 35,478 days
- Data provided on age, race, gender, surgical line, date of surgery, discharge date, and surgery type (scheduled vs. emergency)
- Utilization of post-operative beds varies widely
Discharge Rates

- Discharge rates have positive correlation with utilization

![Bar chart showing discharge rates for different utilization ranges](chart.png)
Utilization Measures

- We compute two measures of utilization
  - Discrete measure – a variable that is 1 when utilization exceeds a given threshold (e.g., 93%), and 0 otherwise
  - Continuous measure – a variable that counts the number of beds in use on each day

- Compare marginal effect of each bed in use vs. a discrete change in discharge probability when utilization exceeds a threshold
Discrete Time Survival Analysis

- Can’t use logistic regression because observations are correlated -- a patient discharged on the fifth day cannot be discharged on the first four days

- Singer and Willet (1993) show how to handle discrete time survival data

- For each day, we record whether or not each patient is discharged, and use this as the outcome variable

- The outcome variable is regressed on our utilization measures and our control variables

- We control for the patient’s age, race, gender, severity, and surgery type
Models and Results

- Model 1: \( \text{logit}(\text{DISCHARGE}) = \text{AGE} + \text{ELECTIVE} + \text{FULL} + \text{CARDIAC SURGERY} + \text{CARDIOLOGY} + \ldots + \text{DONOR SERVICE} + D_1 + D_2 + \ldots + D_{59} + \varepsilon \)

- Model 2: \( \text{logit}(\text{DISCHARGE}) = \text{AGE} + \text{ELECTIVE} + \text{BEDS} + \text{CARDIAC SURGERY} + \text{CARDIOLOGY} + \ldots + \text{DONOR SERVICE} + D_1 + D_2 + \ldots + D_{59} + \varepsilon \)

When the utilization threshold is exceeded, the odds of discharge for any given patient increase. The estimate for Full is positive and significant for threshold above 91.5%.

Each additional bed in use increases the odds that a patient will be discharged. The estimate for Beds is positive and significant.
Observations

- Discharge rates increase as utilization increases, regardless of how utilization is measured.

- Either some patients are held too long and discharged when space is needed, or some patients are discharged too early when utilization is high.

- Our results cannot distinguish between these two explanations.
Research Question 2

- Are patients who are discharged when utilization is high more likely to be readmitted?
  
  Hypothesis: An increase in the discharge rate will lead to some patients with shortened lengths of stay. This will cause an increase in the readmission rate for those patients.
Analysis

- Using the same data set, we apply logistic regression to study the effect that utilization has on the probability of readmission for a specific patient

- We use readmission within 72 hours as our dependent variable

- Hypothesized logistic regression model

\[
\text{logit(READMISSION}_{72}) = \text{AGE} + \text{BLACK} + \text{ASIAN} + \text{HISPANIC} + \text{FULL (or BEDS)} + \text{ELECTIVE} + \text{TRANSPLANT} + \text{TRAUMA} + \ldots + \text{NEURO} + \text{MALE} + \varepsilon
\]
Results

- Model with Full: Controlling for race, age, gender, and the type of surgery, being discharged from a full post-operative unit increases the odds of readmission by a factor of 2.341

- Model with Beds: Controlling for race, age, gender, and the type of surgery, each bed in use at the time of discharge increases the odds of readmission by a factor of 1.008
Utilization-Readmission Relationship

The discharge rate and readmission rate both increase as utilization increases.
Survival Analysis

Over the course of a month, patients discharged from a full hospital are much more likely to be readmitted.
Discussion

- The discharge rate rises when utilization is high
- This corresponds to an increase in the readmission rate
- We conclude that some patients are discharged too soon when utilization is high
- Surgeons have an incentive to clear space for their surgeries
- Mitigation strategy: Use a checklist before discharging a patient—force the surgeon to think about whether the discharge is for the right reason
Conclusions

- Research opportunities in vehicle routing, disaster relief, and healthcare analytics are plentiful.

- The HTSP work presented here will appear in *Optimization Letters*.

- The healthcare analytics work presented here has appeared in *Health Care Management Science* (2011, 2012).

- Thank you!