The Min-Max Multi-Depot Vehicle Routing Problem: Three-Stage Heuristic and Computational Results

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Introduction

• In the Multi-Depot VRP, the objective is to minimize the total distance traveled by all vehicles

• In the Min-Max MDVRP, the objective is to minimize the maximum distance traveled by a vehicle
Introduction

- Min-Max Multi-Depot Vehicle Routing Problem
Introduction

- Min-max objective function
Introduction
Why is the min-max objective important?

• Applications
  ▫ Disaster relief efforts
    • Serve all victims as soon as possible
  ▫ Computer networks
    • Minimize maximum latency between a server and a client
  ▫ Workload balance
    • Balance amount of work among drivers or across time horizon
Introduction
Other considerations

• Fixed number of vehicles available

• Basic model
  ▫ There is no demand associated with the customers

• Capacitated model
  ▫ Customers have demands in terms of quantity

• Service time model
  ▫ Customers require service in terms of time
Literature Review

- Carlsson et al. (2009) proposed an LP-based balancing approach (LB) to solve the Min-Max MDVRP
  - Assignment of customers to vehicles using an LP
  - TSP solved by Concorde
  - These steps are repeated and the best feasible solution is reported
Literature Review

• LB is not expected to work very well when customers are not uniformly distributed

• It is not easy to extend LB to solve problems with customer service times
Solving the Min-Max MDVRP

• We develop a heuristic (denoted by MD)

• MD has three phases

  1. Initialization
  2. Local search
  3. Perturbation
Phase 1: Initialization

- Assign customers evenly to vehicles

\[
\begin{align*}
\min & \quad \sum_{i,j} c_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_{j} x_{ij} = 1 \quad \forall i \\
& \quad \sum_{i} x_{ij} = \left\lfloor \frac{n}{m} \right\rfloor \quad \text{or} \quad \left\lceil \frac{n}{m} \right\rceil + 1 \quad \forall j \\
& \quad x_{ij} \in \{0, 1\} \quad \forall i, j
\end{align*}
\]

- Solve a TSP on each route using the Lin-Kernighan heuristic
Phase 2: Local Search

- Step 1. From the maximal route, identify the customer to remove (savings estimation)
Phase 2: Local Search

• Step 2. Identify the route on which to insert the removed customer (cost estimation)

• Step 3. Try inserting the customer in the cheapest way
  ▫ Successful – go back to Step 1
  ▫ Unsuccessful – try moving another customer

• Step 4. Stop if we have tried to move every customer on the maximal route
Phase 3: Perturbation

- Perturb the locations of the depots
Phase 3: Perturbation

- Solve the new problem
- Set the depots back to their original positions
- Solve the problem and update the solution
- Repeat the process until there is no improvement for five consecutive perturbations
Phase 3: Perturbation

- The angle of subsequent perturbation depends on the angle of the previous perturbation.
Computational Results

• 20 test problems
  ▫ 10 to 500 customers
  ▫ 3 to 20 depots
  ▫ Problems have uniform and non-uniform customer locations

• MD used an Intel Pentium CPU with 2.20 GHz processor

• Code for LB required a 32-bit machine (Intel Core i5 with 2.40 GHz processor)
Computational Results

Problem MM8 (3 depots, 200 customers, 2 vehicles)
Computational Results

Problem MM8 (3 depots, 200 customers, 2 vehicles)
Computational Results

Uniform Customer Locations

- MD outperforms the LB-based heuristic by 12.5% on average

<table>
<thead>
<tr>
<th>Identifier</th>
<th>LB Objective</th>
<th>Time (s)</th>
<th>MD Objective</th>
<th>Time (s)</th>
<th>Improvement (%)</th>
<th>VRPH*</th>
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<tbody>
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<td>MM2</td>
<td>149.225</td>
<td>38.2</td>
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Computational Results
Non-uniform Customer Locations
• MD outperforms the LB-based heuristic by 19.0% on average

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<th>LB Time (s)</th>
<th>MD Objective</th>
<th>MD Time (s)</th>
<th>Improvement (%)</th>
<th>VRPH</th>
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Conclusions

• On the 20 test problems, MD outperforms the LB-based heuristic by 13.2% on average

• In future work, we want to investigate the quality of the MD solution when applied to the service time model

• We also hope to enhance MD in order to produce even better solutions