Plowing with Precedence

A Variant of the Windy Postman Problem

Benjamin Dussault, Bruce Golden, Edward Wasil, and Chris Groer

Overview

- The Chinese Postman Problem (CPP)
- The Windy Postman Problem (WPP)
- Plowing with Precedence
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    - Introduction
    - Problem Statement
    - Problem Formulation
    - Solution Methodology
    - Results
    - Conclusions

The Chinese Postman Problem (CPP)

- Consider a graph $G=\{V,A\}$ where
  - $V=\{v_i\}$
  - $A=\{(v_i,v_j) \mid v_i,\ v_j \in V\}$
  - c_{ij} = Cost of traversing on arc (v_i,v_j)
  - $c_{ij} = c_{ji}$
- Goal: To construct a least-cost tour which visits all arcs in A at least once

The Windy Postman Problem (WPP)

- A close variant of the Chinese Postman Problem
- The graph is “Windy”; it is harder to traverse in one direction on an arc as opposed to the other
- Goal: To construct a least-cost tour which visits all arcs in A at least once
- Key Difference: Costs are not symmetric

Solution Methodology of CPP and WPP

- Crucial observation: If graph is Eulerian, then an optimal tour can readily be obtained using Fleury's Algorithm
- It is therefore sufficient to convert the instance graph to a Eulerian graph in an optimal way
- Possible methods
  - Integer Programming
  - Add least-cost paths between odd-degree nodes

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Literature Review

- Arc Routing is well studied. There are many summaries:
  - Eiselt et al. (1995a, 1995b)
  - Assad and Golden (1995)
  - Dror (2000)
- Perrier et al. (2006, 2007) provide a four-part summary of winter road maintenance covering:
  - System Design
  - Models and Algorithms
  - Vehicle Routing and Depot Location
  - Vehicle Routing and Fleet Sizing

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Introduction

- Variant of the Windy Postman Problem
- Non-symmetric costs correspond with the difficulty of plowing uphill
- Once a street is plowed, the cost of subsequent traversals is significantly less
  - Requires two new costs for each arc: the cost of deadhead in each direction
  - Introduces the concept of precedence: the cost of a street now depends on wether it has been traversed already

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Introduction

- The concept of precedence requires a fundamentally different solution methodology than that used in previous WPP literature
- A Eulerian graph yields many Eulerian tours
  - Equivalent in WPP
  - Not equivalent in Plowing with Precedence

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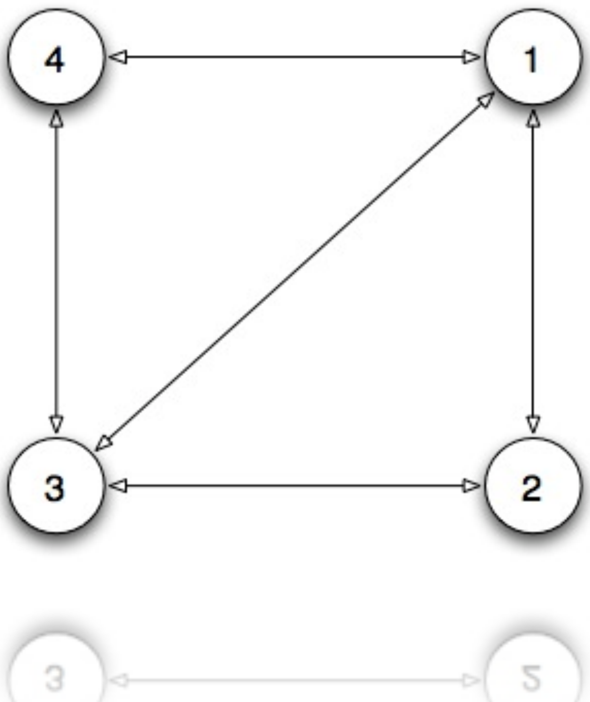
Introduction

Deadhead costs = 1



Original Instance

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Introduction

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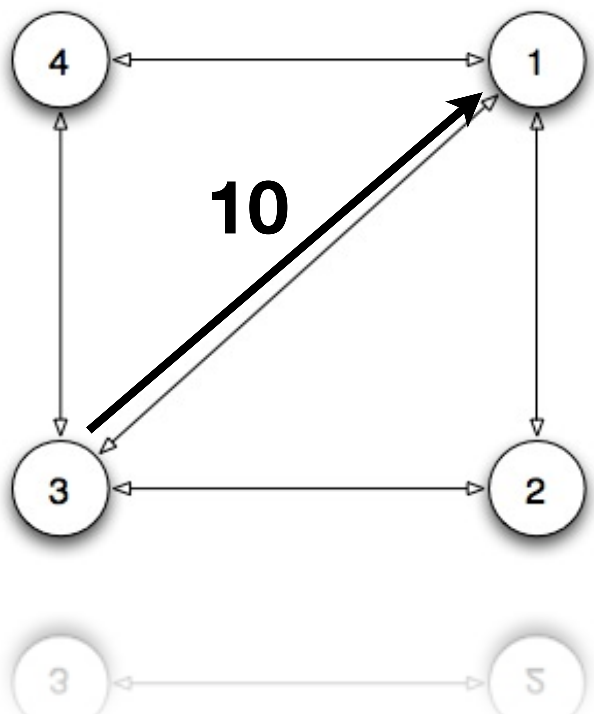

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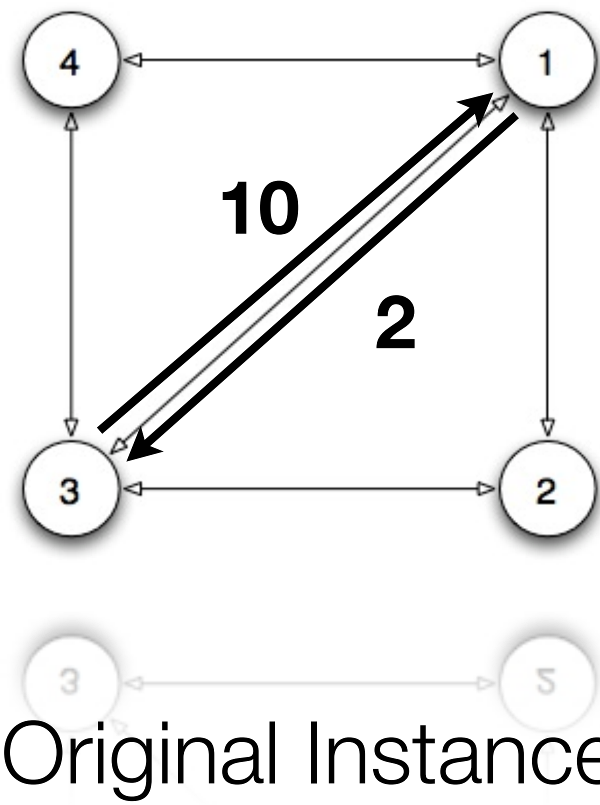

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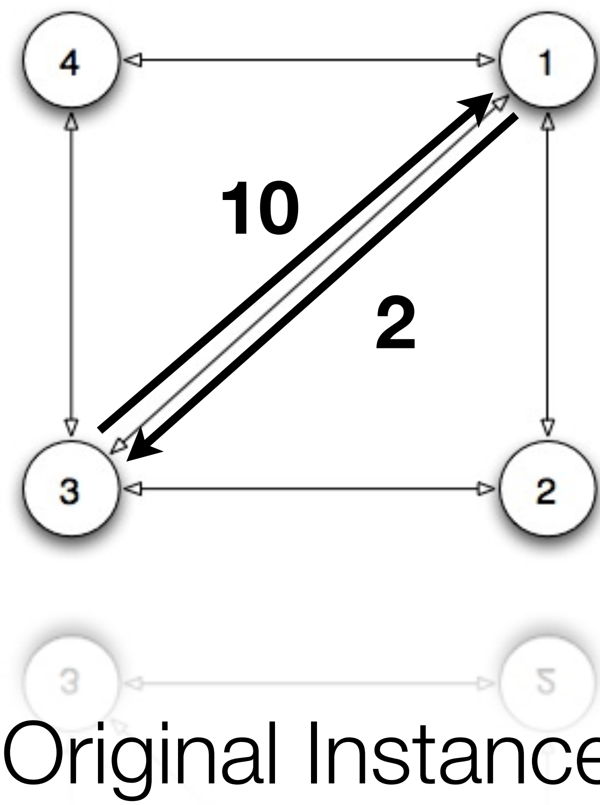

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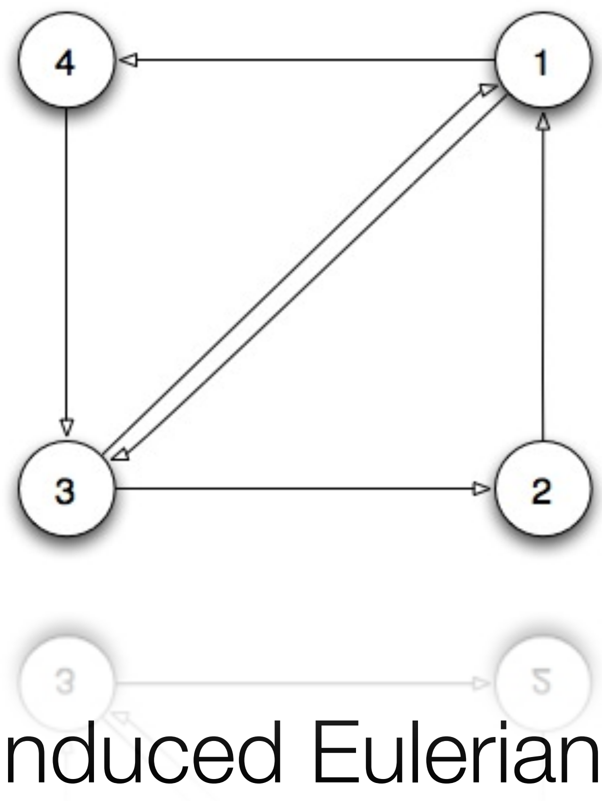

Induced Eulerian Graph

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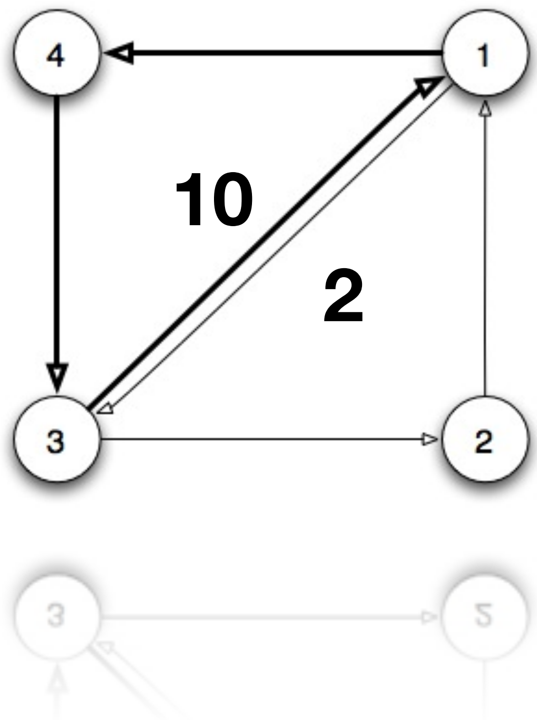
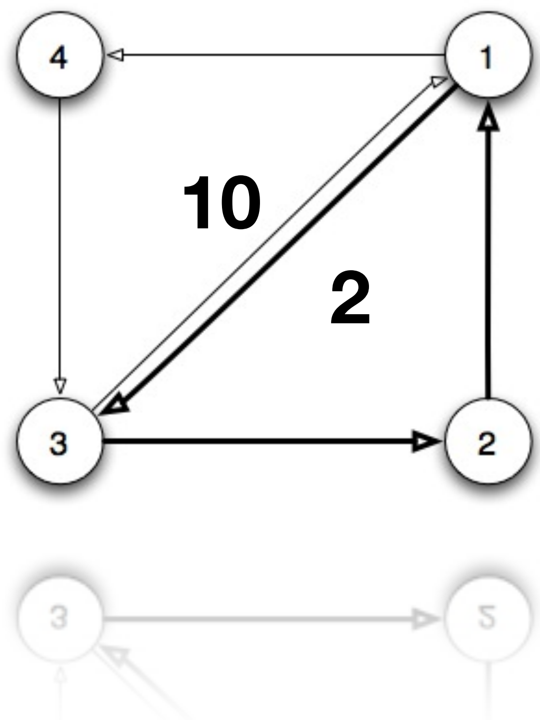
Introduction

- Multiple tours:
  - {1,4,3,1,3,2,1}
    - Travels arc (3,1) before (1,3)
  - {1,3,2,1,4,3,1}
    - Travels arc (1,3) before (3,1)





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Problem Statement

- ❖ Consider a graph $G=\{V,A\}$ where
  - $V=\{v_i\}$
  - $A=\{(v_i,v_j) \mid v_i,\ v_j \in V\}$
  - c_{ij}^{+} = Cost of plowing on arc (v_i,v_j)
  - c_{ij}^{-} = Cost of deadheading on arc (v_i,v_j)
  - $c_{ij}^{+} >> c_{ji}^{+} >> c_{ij}^{-} \geq c_{ji}^{-}$
- ❖ Goal: To construct a least-cost tour which visits all arcs in A at least twice (once for each side of the street) and begins and ends at a depot (required to incorporate precedence)

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Problem Statement

- Non-directed arcs allow plowing against the flow of traffic
- Good solutions will attempt to plow downhill both times
- Allows for the intriguing possibility of:
  - Plowing downhill
  - Then deadheading uphill
  - Then plowing downhill

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Problem Formulation

- Requires an index *t* to incorporate precedence
- Essential elements:
  - x_{ijt} = 1 if plow (*i,j*) at time t, 0 otherwise
  - y_{ijt} =1 if deadhead (*i,j*) at time t, 0 otherwise
  - φ_{ijt} =1 if (*i,j*) is first plowed at time t, 0 otherwise
- Essential constraints:
  - Tour continuity
  - Forbid deadhead on (*i,j*) until (*i,j*) or (*j,i*) is plowed
- Large number of variables and constraints (~8000 and 19000 respectively for an instance with 10 arcs and 7 nodes)

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Solution Methodology

- ❖ Construct solution framework using integer programming
  - Objective seeks to minimize framework tour cost
  - Solution serves as a lower bound
- ❖ Use solution framework to construct initial solution using Fleury's Algorithm
- ❖ Perform local search on obtained solution
  - Reinitialize and repeat local search
- ❖ Prune obtained solution to obtain final solution

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Solution Methodology - Solution Framework

- Adapt IP formulation for Windy Postman Problem
  - Ignores the concept of precedence, otherwise solves the problem
  - Objective function, which seeks to minimize ideal tour cost, serves as a useful lower bound
- Essential variables:
  - x_{ij} = the ideal number of times (i,j) is plowed
  - y_{ij} = the ideal number of times (i,j) is deadheaded
- Essential constraints:
  - Plow each street twice
  - Degree matching for each node

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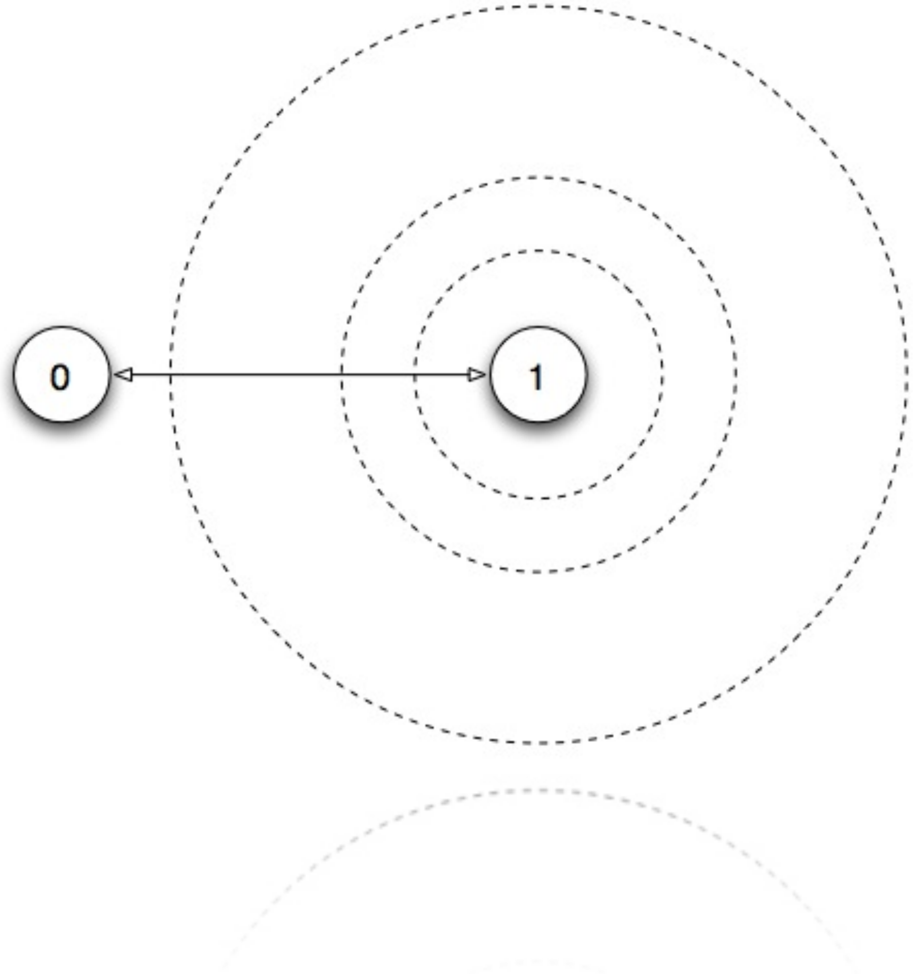
Solution Methodology - Solution Framework

- It is possible that no tour will yield the objective function of the solution framework
- Let the cost of (0,1) be 10 and the cost of (1,0) be 2
- Let the deadhead costs be 1



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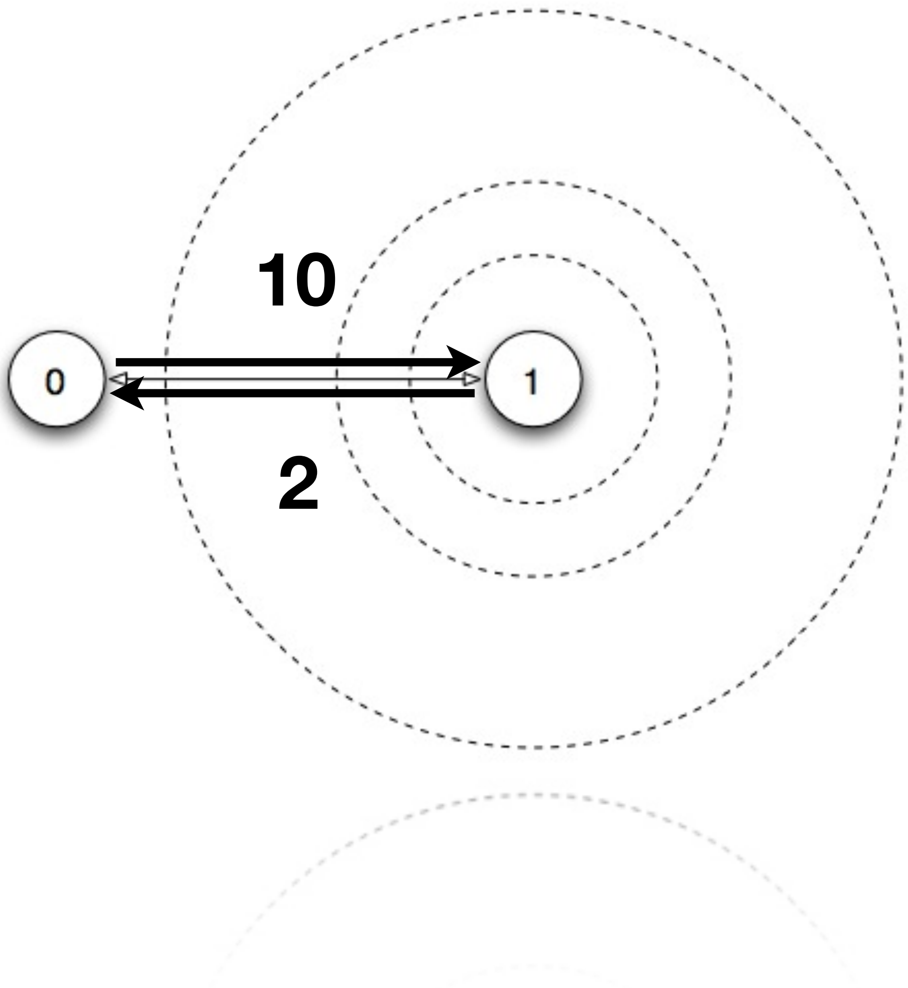

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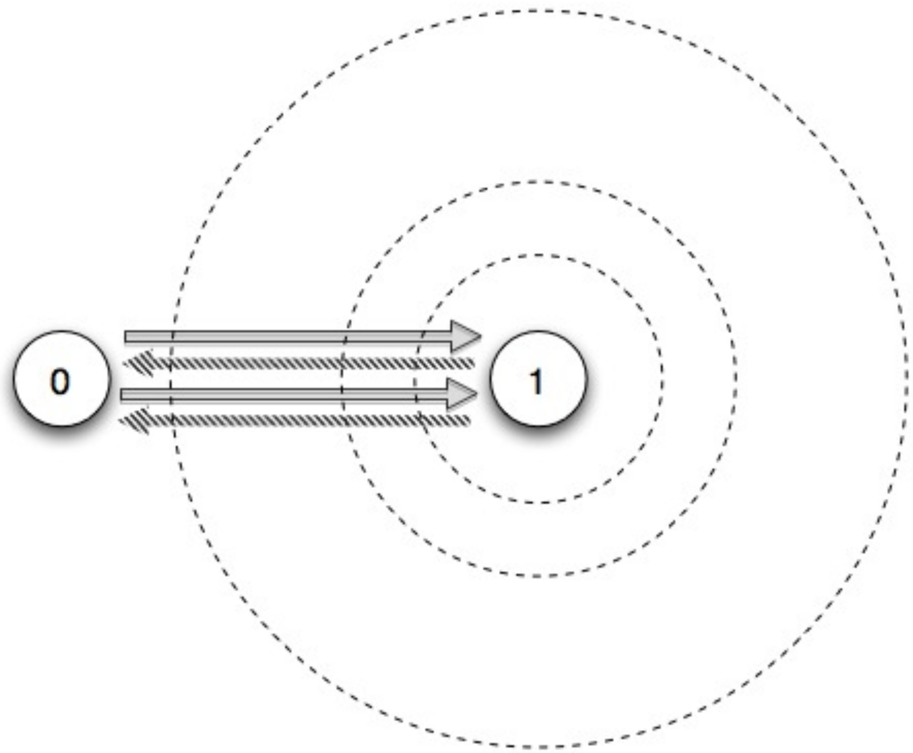

Solution Framework

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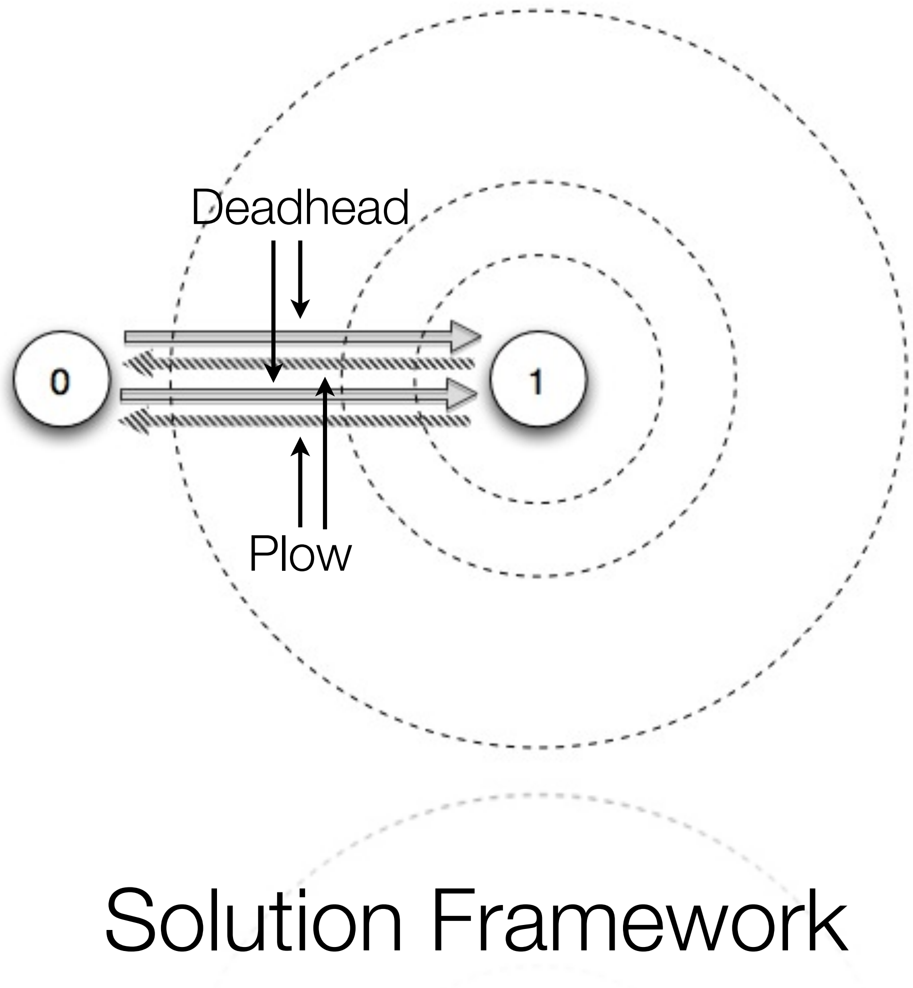

Solution Framework

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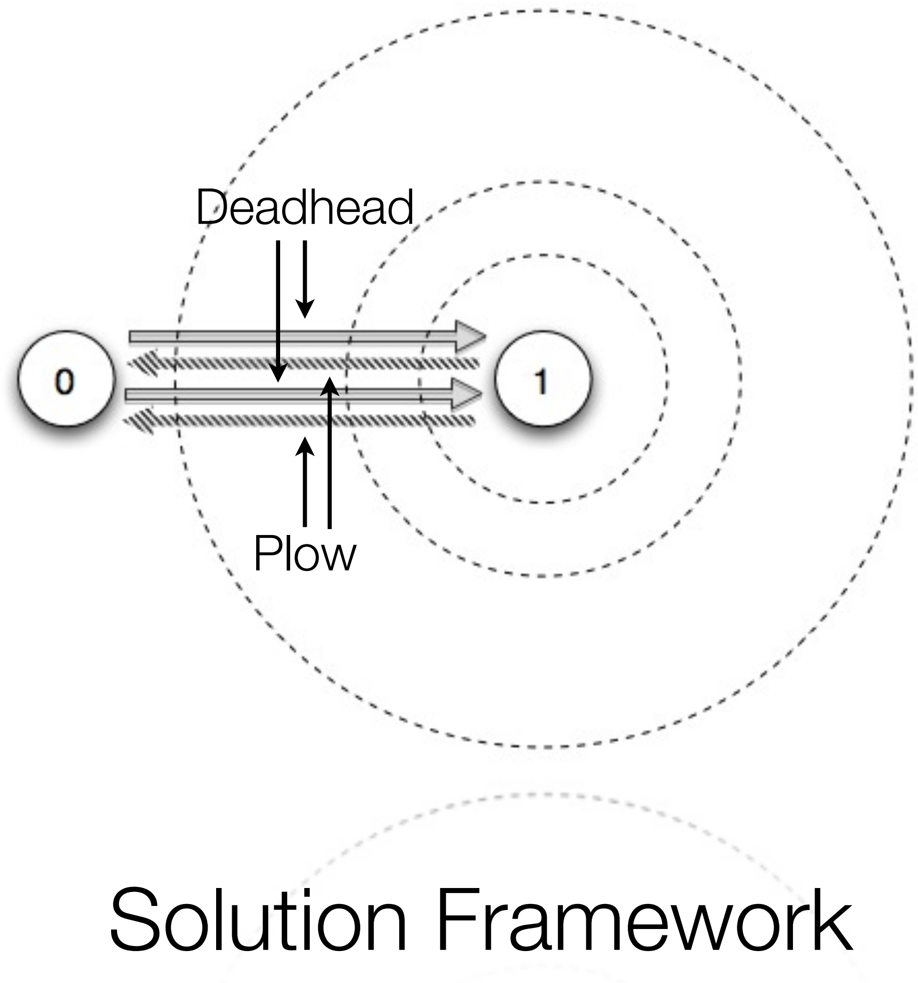
Solution Methodology - Solution Framework



Solution Framework

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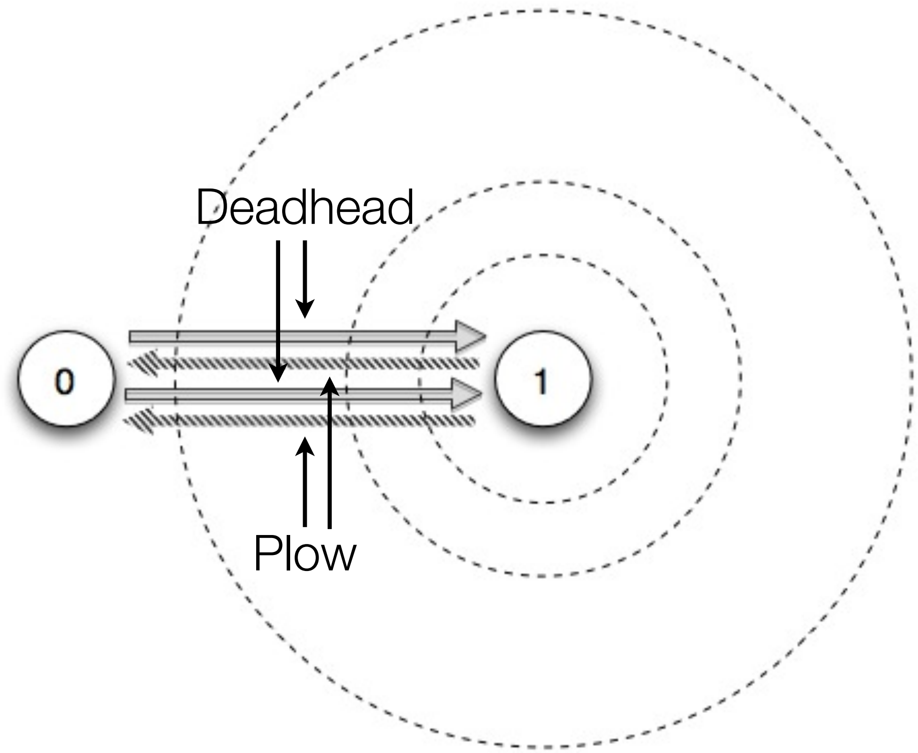
Solution Methodology - Solution Framework

Solution framework seeks to plow downhill twice

Plowing uphill is unavoidable

Solution framework has objective value of 6

Optimal tour {0,1,0} has cost 12



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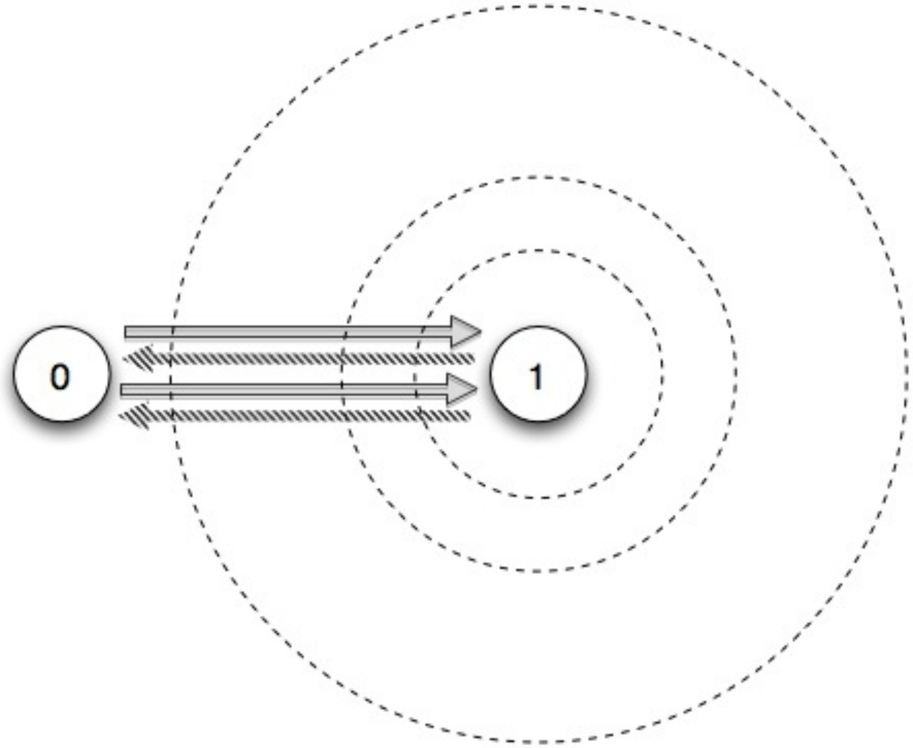

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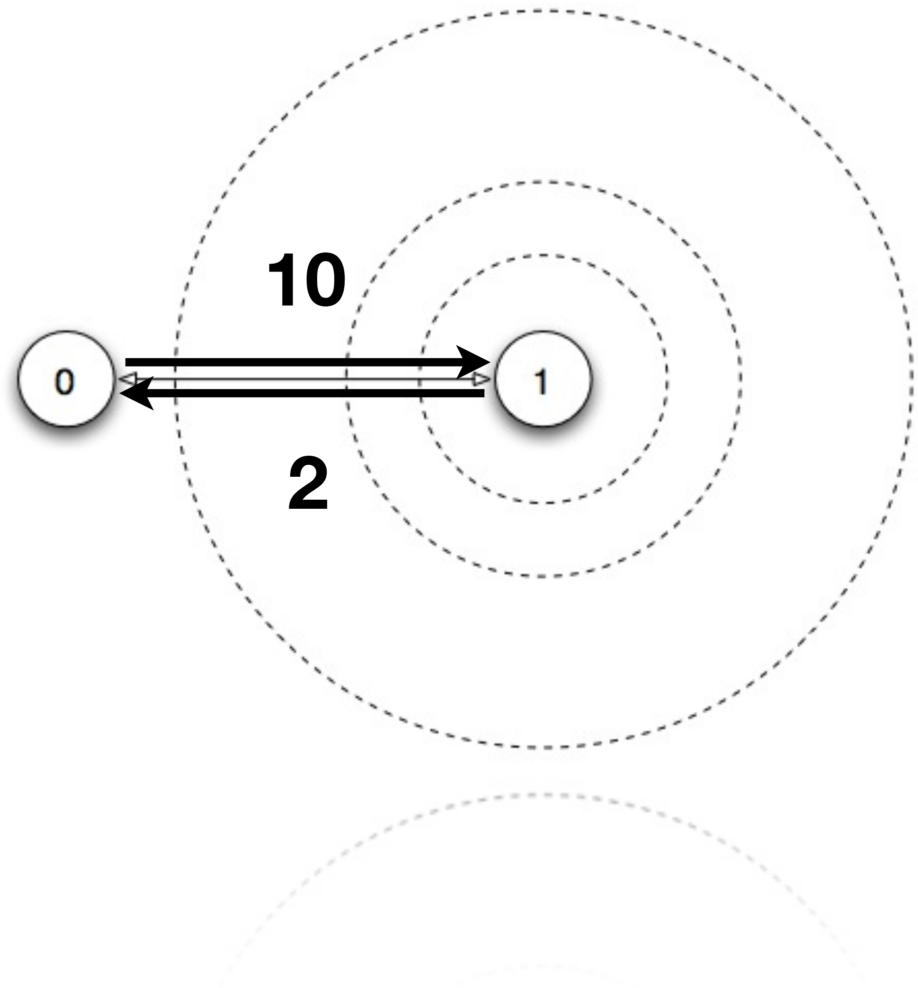

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Solution Methodology - Initial Solution

- A tour can be obtained from solution framework by using Fleury's Algorithm
- This tour is guaranteed to traverse (and hence plow) each street twice
- Not guaranteed to have a cost that is the same as the lower bound of the solution framework (previous example)
- Seek to modify tour using a local search heuristic

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Solution Methodology - Local Search

- We seek to explore the set of all Eulerian tours that obey the solution framework
- Local search searches “nearby” tours in an attempt to find a better one
- Requires:
  - Definition of neighborhood - defines what nearby is
  - Fitness function - gives the quality of a tour
    - In our case, the fitness is the cost of the tour

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Solution Methodology - Local Search

- Solution Fitness:
- For each arc, decide to plow based on the following decision tree:

  if arc has been plowed twice
  → then don’t plow
  else if arc hasn’t been plowed at all
  → then plow
  else if going downhill
  → then plow
  else if tour isn’t going downhill later
  → then plow
  else don’t plow

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Solution Methodology - Local Search

- All Eulerian tours can be decomposed into cycles
- Definition of neighborhood around a solution s, N(s): the set of all tours that can be obtained by a combination of the following moves:
  - Cycles in the tour are permuted
  - Cycles in the tour are reversed

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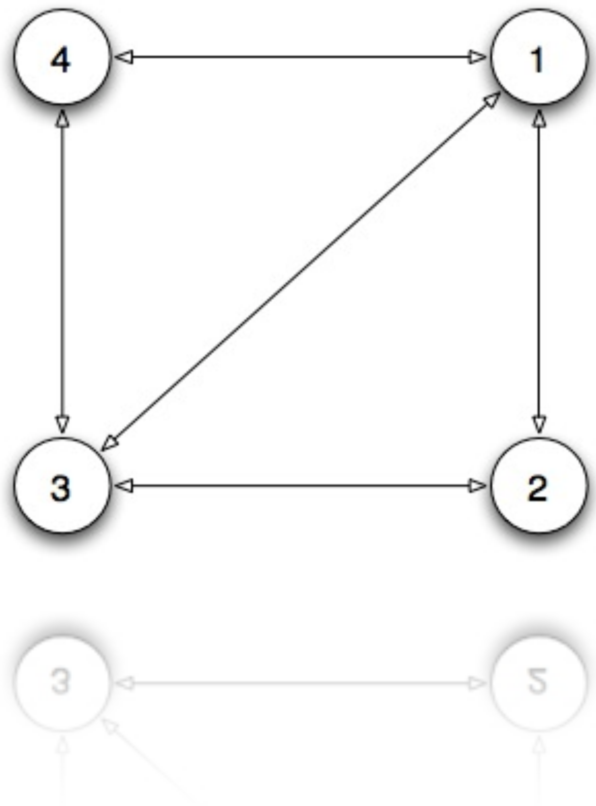
Solution Methodology - Local Search

{1,2,3,1,2,3,4,1,3,4,1}



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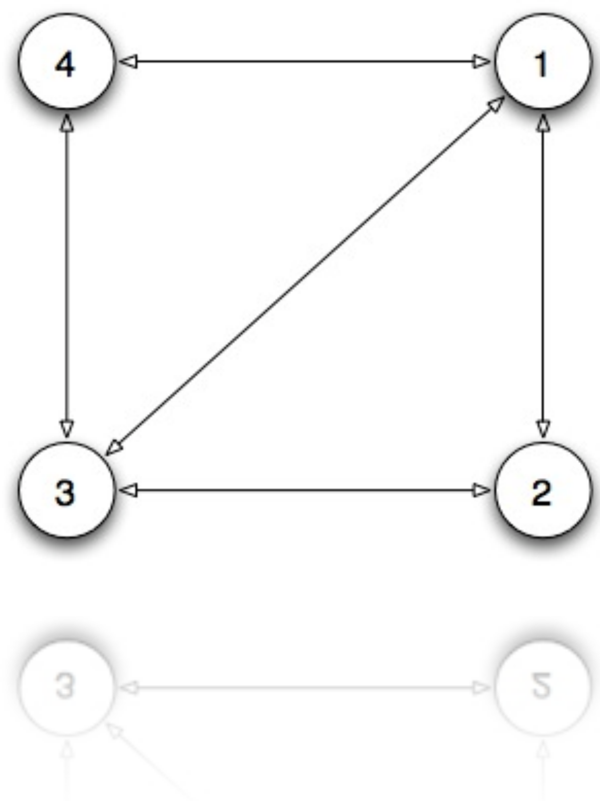

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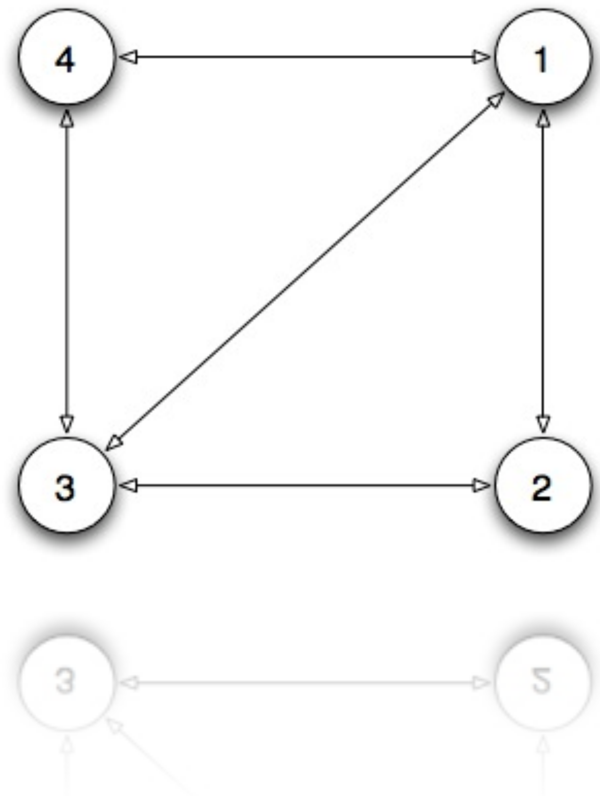

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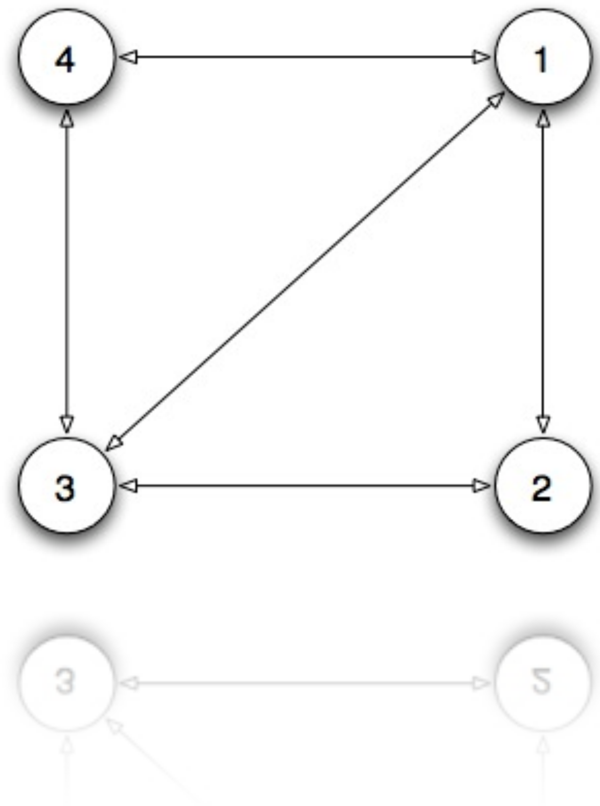

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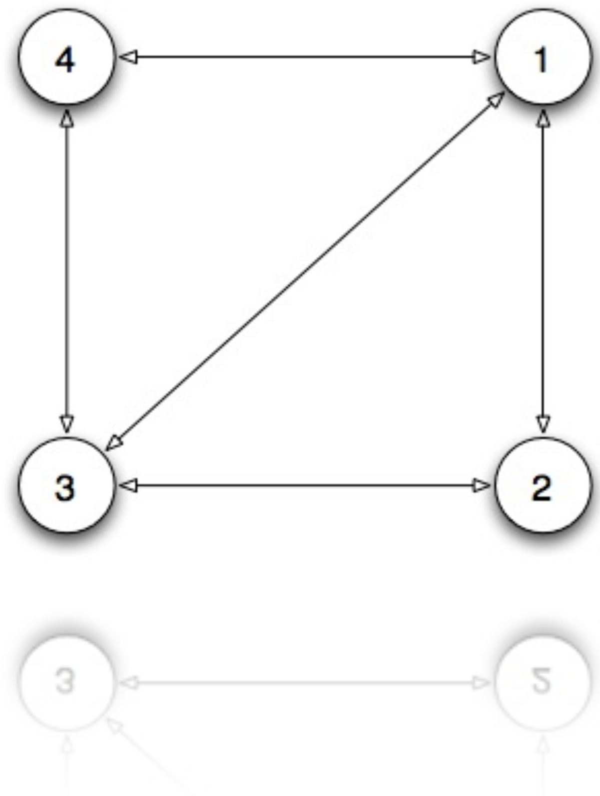

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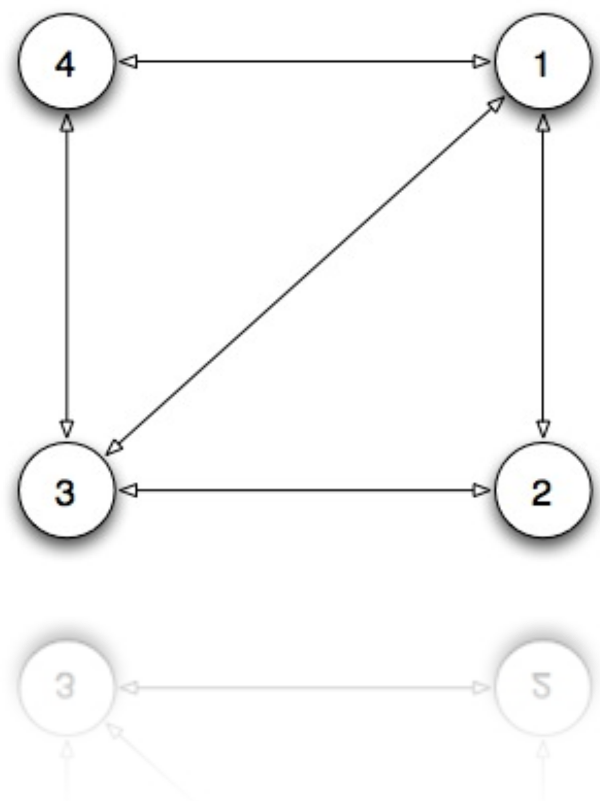

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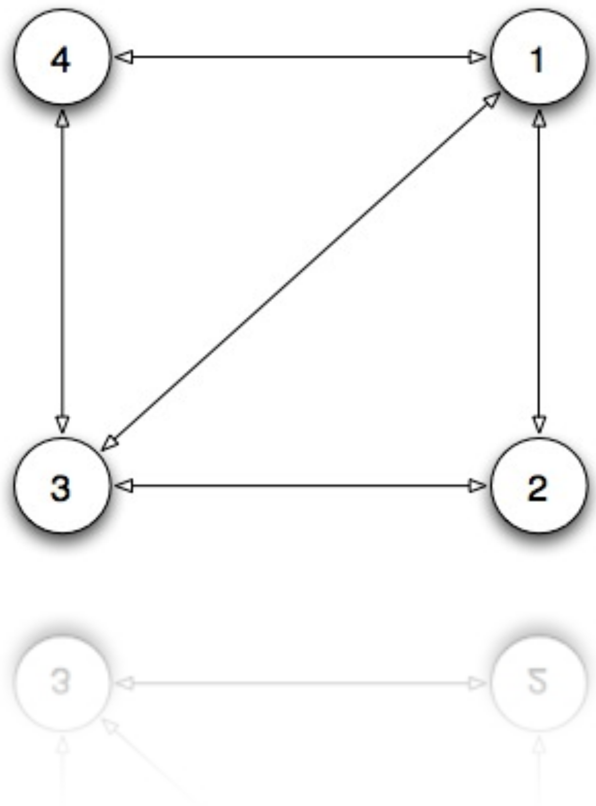

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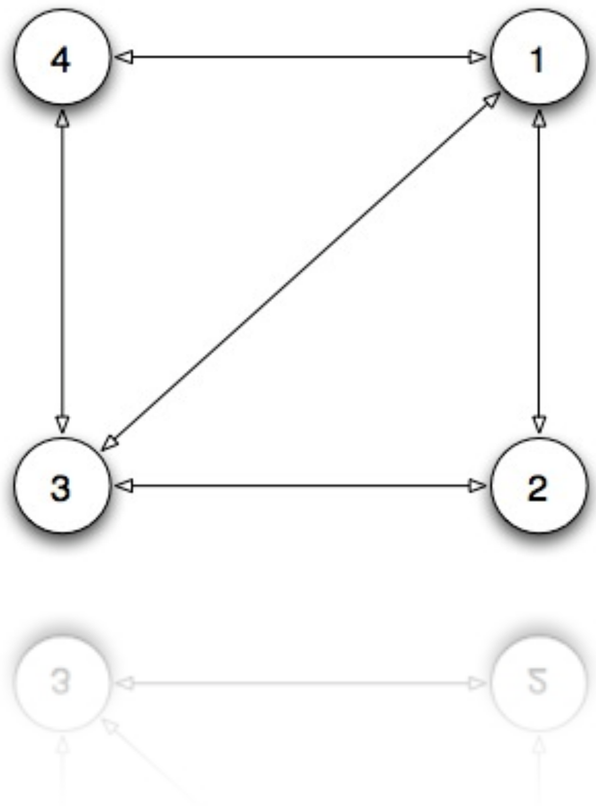

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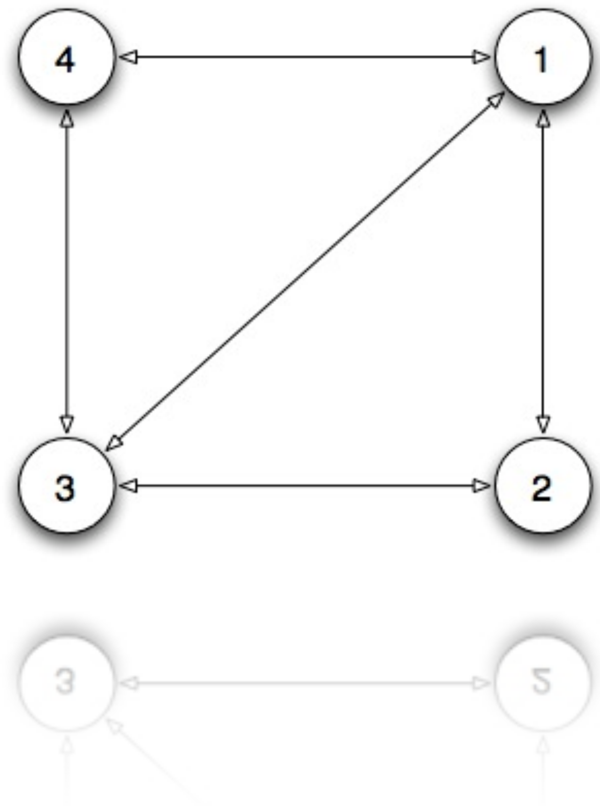

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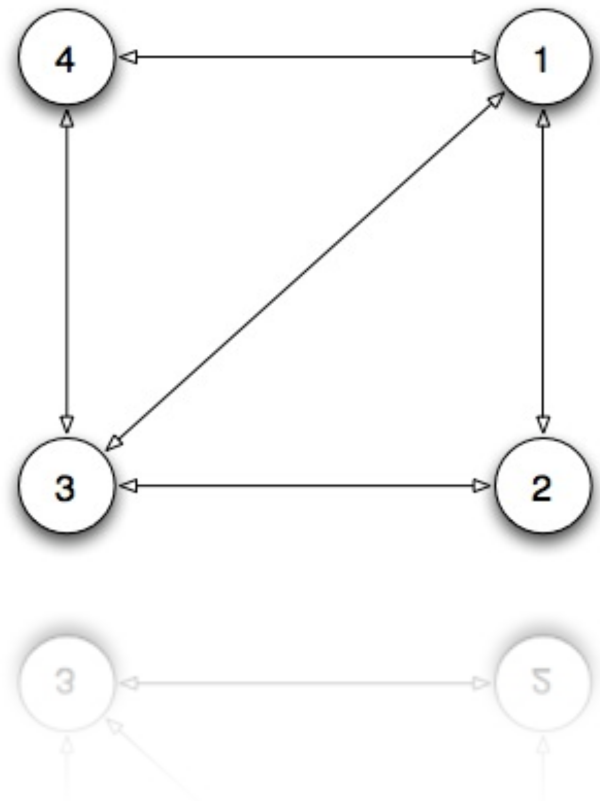

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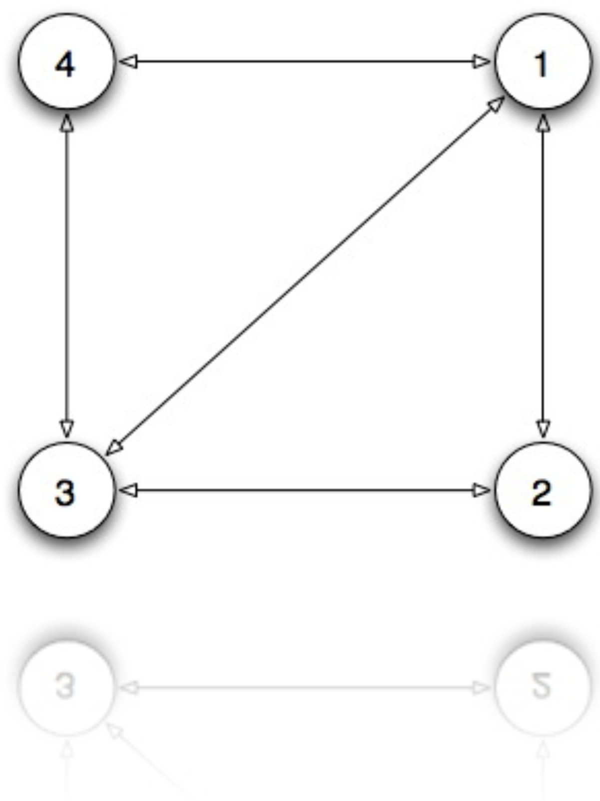

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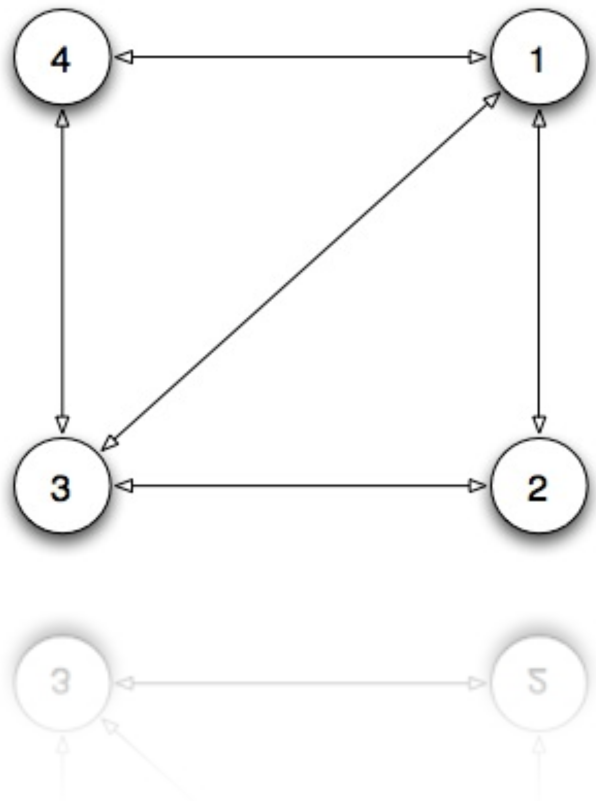

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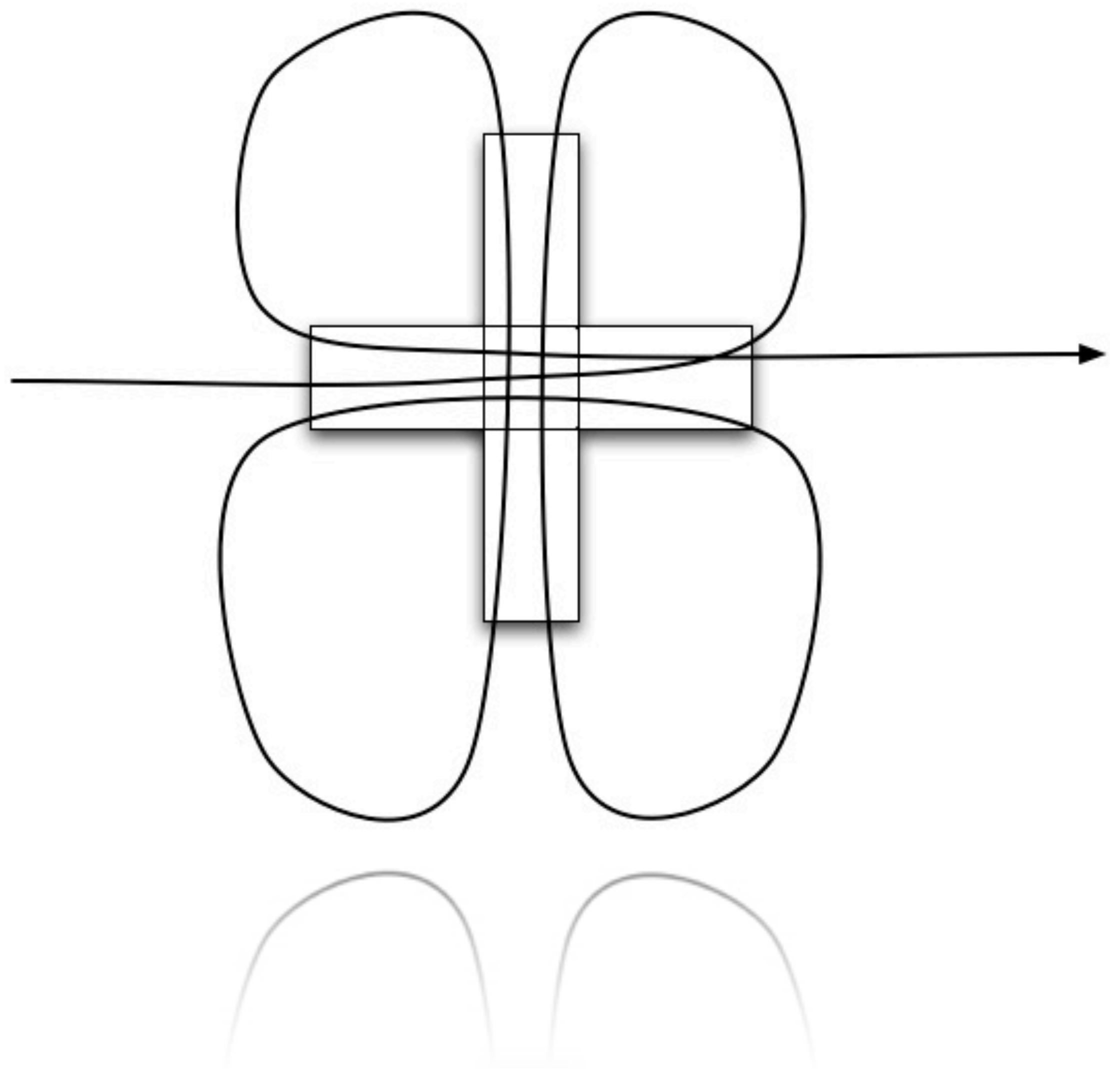
Solution Methodology - Local Search

- The number of permutations is large: n! for n cycles
- To limit the size of the neighborhood, if n>4, we limit the set of permutations to 4!+n for linear growth
- Most intersections have four or fewer cycles



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Solution Methodology - Reinitialization

- Local search is deterministic and dependent on the initial solution
- We reinitialize to obtain new initial solutions to run the local search procedure on
- This is done by permuting cycles around different nodes randomly a large number of times
- The best solution obtained by 15 runs of the local search and reinitialization combination is retained

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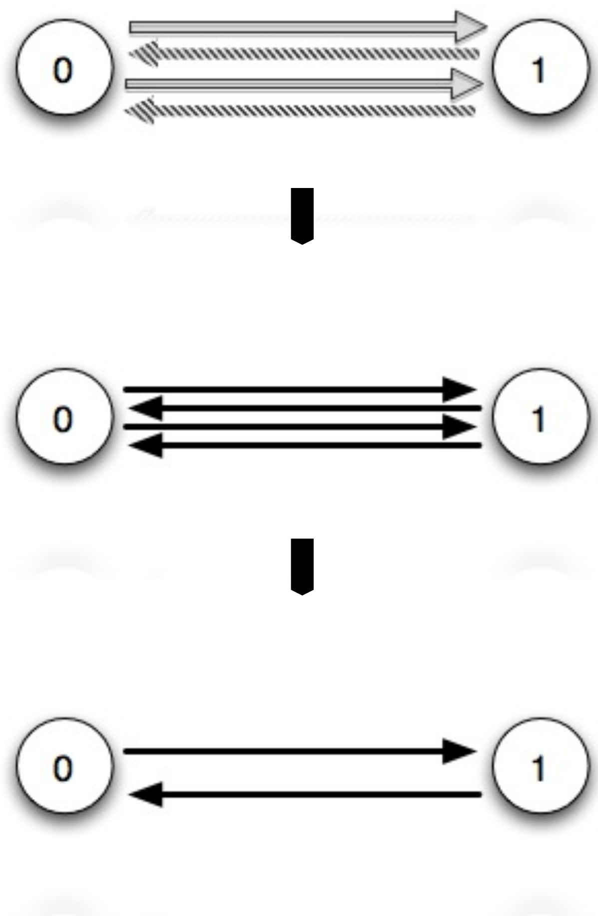
Solution Methodology - Pruning

- It is possible that a tour will have cycles that consist of entirely deadhead
- These cycles can be pruned to obtain a lower-cost tour that still plows each street twice
- Pruning is done at the end of local search + reinitialization phase



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Results

- We test our algorithm on a bank of modified Windy Rural Postman Problems presented in Corberan et al.
  - Remove Rural concept
  - Existing costs are interpreted as plowing costs
  - Randomly generate deadhead costs
- Instances are characterized by:
  - Number of nodes (7-196)
  - Number of arcs (10-316)
  - Average cost deviation - average discrepancy in cost between plowing up and plowing down (4%-80%)

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Results

- Our IP formulation is too large to solve all but the smallest of instances
- We therefore compare against the lower bound given by the solution framework
  - If we obtain the lower bound, then we know we have the optimal solution
- Our algorithm performs very well
  - Obtains optimal solution on more than 50% of the instances
  - Averages 0.2% deviation from the lower bound

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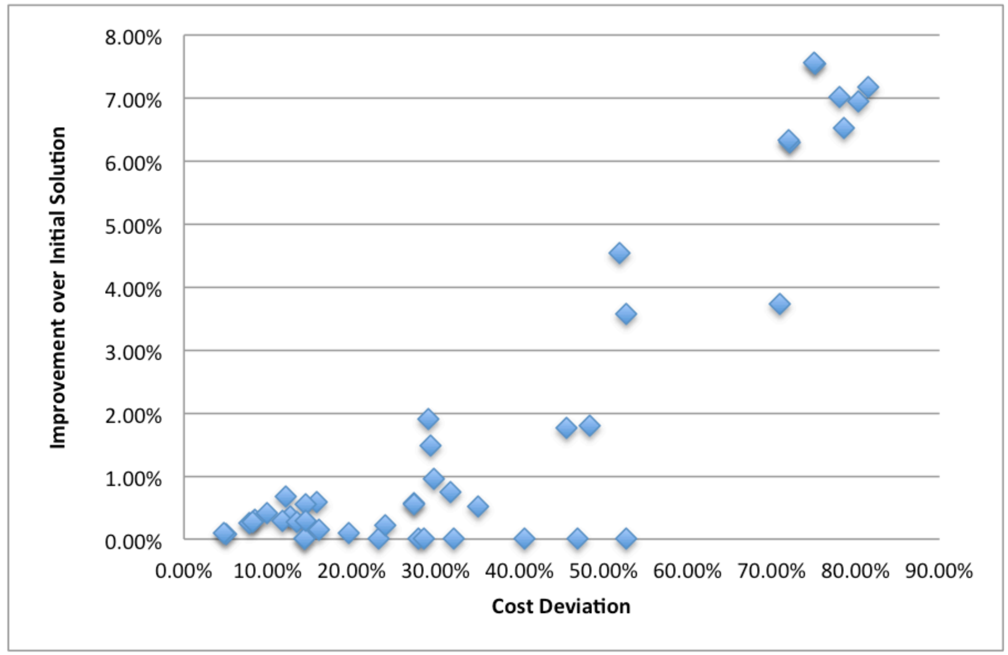
Results

- Compare final solution cost against the initial solution cost
- 1.8% average improvement
- Measure percentage improvement vs. Average cost deviation



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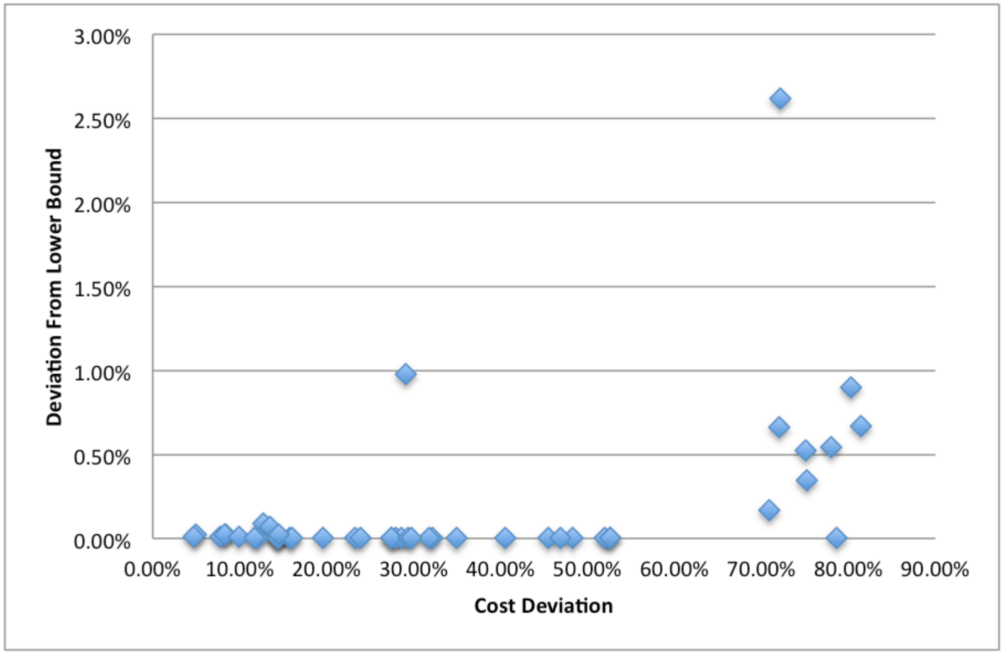
Results

- Cost deviation is largest driving factor in deviation from lower bound
- 0.17% average deviation from the lower bound
- Deviation from LB increases as cost deviation increases
- Want to investigate further



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Results

- We took two large instances and constructed several new instances that:
  - Preserved the same graph
  - Average cost deviation ranged from 10% to 70%
- Compare the effects of average cost deviation on:
  - Running Time
  - Percentage Improvement
  - Deviation from Lower Bound

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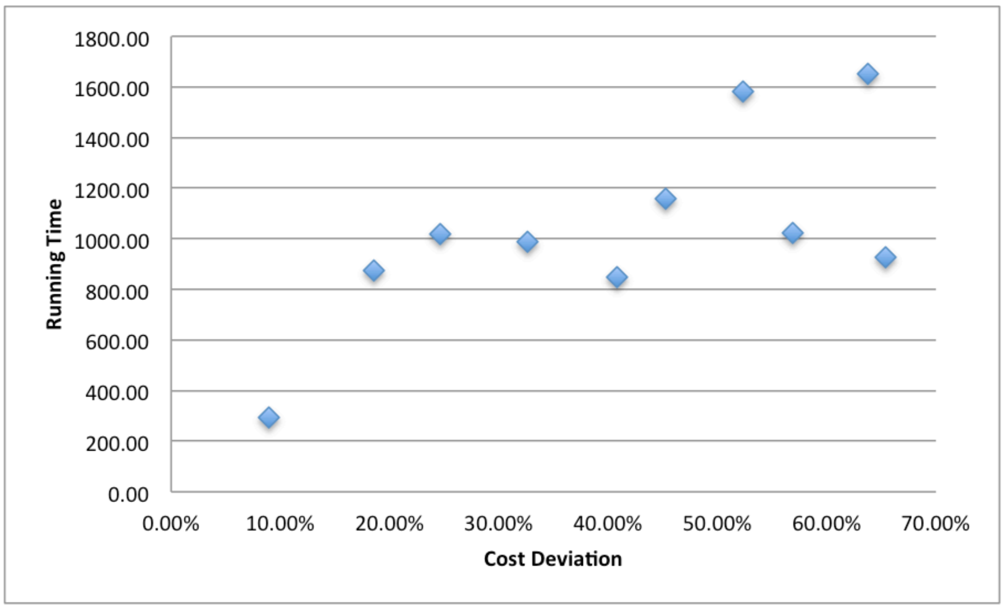
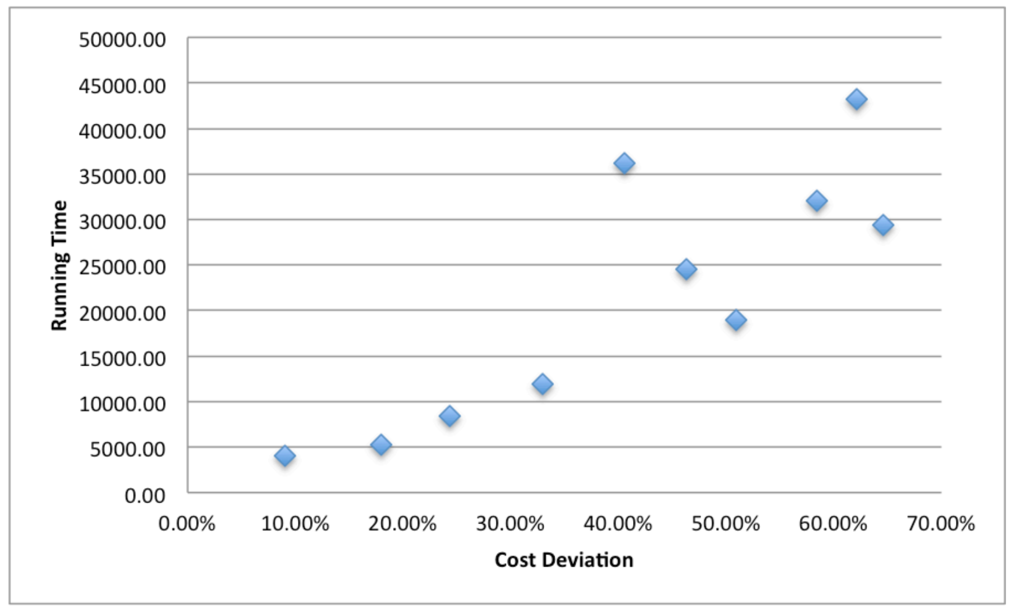
Results

Running Time vs. Average Cost Deviation





Instance A3101

Instance M3101

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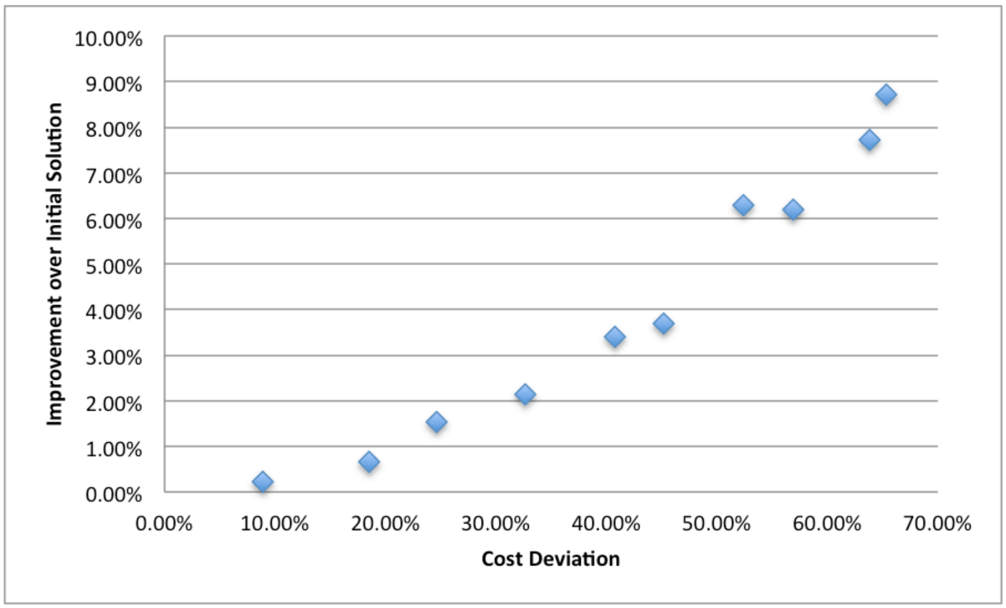
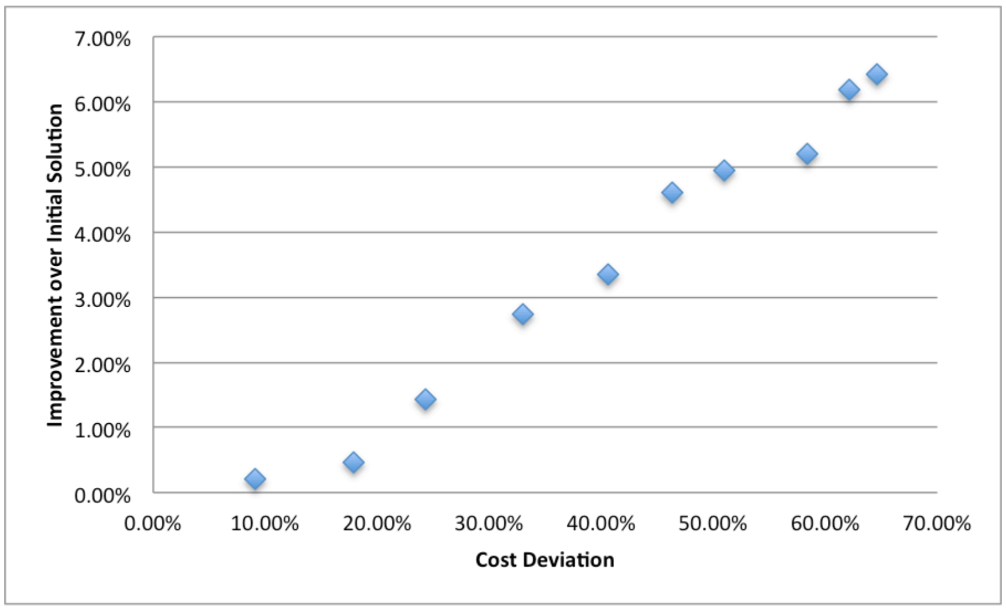
Results

Percentage Improvement vs. Average Cost Deviation





Instance A3101

Instance M3101

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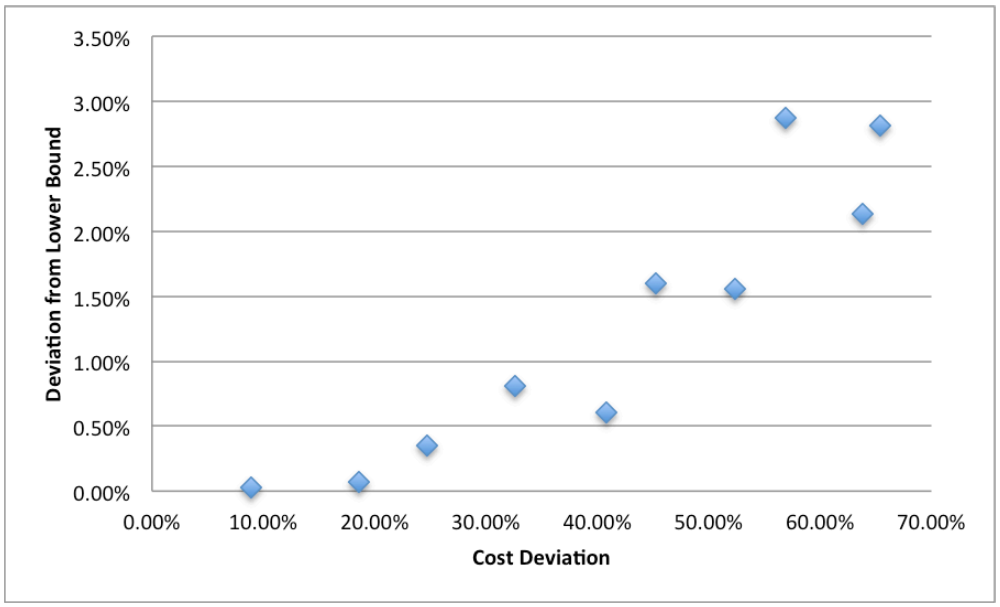
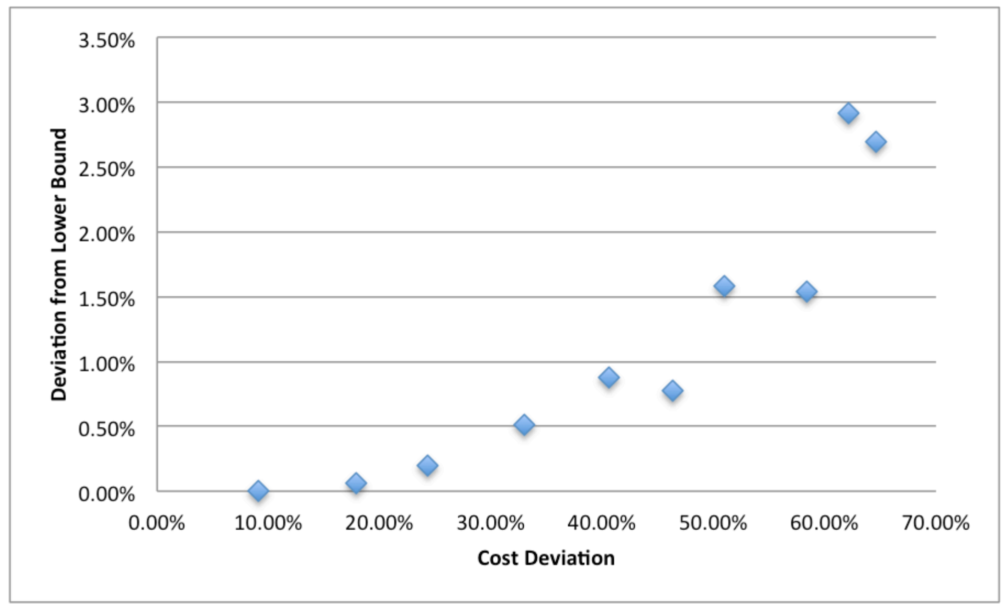
Results

Deviation from Lower Bound vs. Average Cost Deviation



Instance A3101



Instance M3101

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Conclusions

- Introduced a variant of the WPP
- Addresses the practical consideration that deadheading a street after plowing is less costly than plowing the street
- Introduces the concept of precedence to postman problems
- Obtain very good results, producing solutions that are, on average, within 0.79% of the lower bound
  - Solutions are very often optimal
- Observed increases in running time, percentage improvement, and deviation from the lower bound as a function of the average cost deviation

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Conclusions

- Future work:
  - Generalize the concept of precedence: Let the cost of traversal be a general function of the number of times traversed
  - Add multiple plows: When one plow plows a street, other plows are able to deadhead that street