The exam consists of three problems. Please show all work and give explanations for all answers since the reasoning behind your answer is as important as the final answer.

1. (35 points)

(a) Consider a current loop of radius "R" with current I flowing in the clockwise direction as shown. Indicate the direction of the magnetic field (in the plane of the paper) inside and outside of the loop.

(b) Calculate the strength of the magnetic field at the center of the loop. Evaluate $B$ at this location for $I = 1\, \text{A}$ and $R = 1\, \text{cm}$.

(c) Sketch the electric field $\mathbf{E}$ near the center of the loop as $I$ is reduced. Explain.

(d) Consider a circuit with a resistor, capacitor and inductor as shown. The switch is closed at $t = 0$. Just after the switch is closed, what current flows through the capacitor? Inductor?

(e) At late time what currents flow through the inductor and capacitor. What is the voltage drop across the capacitor at this time?
2. (30 points) An inductor is constructed like a co-axial cable. An inner conductor of radius $R$ carries a current $I$ into the page. The current is distributed uniformly across the conductor. A hollow outer conductor of negligible thickness carries the reversed current $-I$. The length of the inductor is $d$. Assume that $R \ll d$.

(a) Sketch the magnetic field $B$ produced by this configuration of current.

(b) Calculate the magnetic field everywhere assuming that end effects can be neglected.

(c) Calculate the magnetic energy stored in the device and its self-inductance $L$.

3. (35 points) Consider a wire loop of width $w$, length $L$, mass $m$ and resistance $R$. The loop is dropped into a half plane magnetic field $B$ with gravity $g$ as shown. Assume that the loop is constrained to remain in a vertical plane and that $L$ is very large.

(a) In which direction will the current flow in the loop as the loop enters the magnetic field. Why?

(b) For a given velocity $v$ of the loop calculate the current $I$.

(c) Calculate all forces acting on the loop. What is the direction and magnitude of the net force acting on the loop at a specified velocity $v$.

(d) Qualitatively describe the time dependence of the velocity of the loop after it is dropped assuming that the low edge of the loop enters the magnetic field just as it is dropped. Calculate the velocity at late time but prior to the top of the loop entering the magnetic field. What happens after the top of the loop enters the magnetic field.

(e) Calculate the rate of dissipation of energy in the resistor at late time but prior to the top of the loop entering the magnetic field. What is the source of the energy dissipated in the resistor?
b) Bat loop centered

Use $dB = \frac{M_0 I}{4 \pi a} \frac{dl \times \hat{r}}{r^2}$

$\Rightarrow$ points into page $dl \perp r$

$dB = \frac{M_0 I}{4 \pi a} \frac{dl}{r^2} = \frac{M_0 I}{4 \pi a} \frac{dl}{R^2}$

$\Rightarrow R$ same for all points on wire

$B = \frac{M_0 I}{4 \pi a R^2}$

$\oint dl = \frac{M_0 I}{4 \pi R^2}$

$B = \frac{M_0 I}{2R}$

$B = \frac{2 \times 10^{-7} Tm}{A \times (0.01) m} \frac{1A}{A}$

$B = 2 \pi \times 10^{-5} T$
c) Reduce current

From Lenz' law, B in center points into page and decreases in time. A clockwise current would act to increase B inside the small loop shown.

⇒ clock wise \( E_m \).

d) Loop rule; left loop,

\[ E - I_R R - \frac{Q}{C} = 0 \]

no change on cap.

\[ I_R = I_C = \frac{E}{R} \]

just after switch closed

\[ I_L = 0 \Rightarrow \text{no time to pump energy in inductor.} \]

de) Late time \( \Rightarrow \) assume steady state

\[ \frac{dI_L}{dt} = 0 \Rightarrow E_L = 0 \Rightarrow E_C = 0 \]

since are in parallel

\[ I_L = I_R = \frac{E}{R} \Rightarrow I_C = 0 \]
Clockwise $B$ for $r < R$. $\mathbf{B} = 0$ for $r > R$

Use Ampere's law because of symmetry. Consider loop of radius $r_0$.

b) $\oint \mathbf{B} \cdot d\mathbf{l} = B \cdot 2\pi r = \mu_0 I = \mu_0 J \pi r^2$

Inside inner conductor constant current density $J = \frac{I_{tot}}{\pi R^2}$

$B_2 \pi \Delta = \mu_0 \frac{I}{\pi R^2} \Delta r^2$

$$B = \frac{\mu_0 I r}{2\pi R^2} \quad r < R$$

For $r > R$

$B \cdot 2\pi r = \mu_0 (I + (-I)) = 0$

$\Rightarrow \mathbf{B} = 0$

c) Magnetic energy

$W_B = \int_S \mathbf{B} \cdot \mathbf{v} \cdot \frac{B^2}{2\mu_0}$

Since $B$ depends on $r$, must choose $d\mathbf{v}$ that has small size in radial direction.

$d\mathbf{v} = radrd\theta$

$$W_B = \int_0^\pi \int_0^R r^2 \pi rd\theta \frac{\mu_0 I^2 r^2}{4\pi^2 R^4}$$
\[
\omega_B = \frac{d \mu_0 I^2}{4 \pi R^4} \int_0^R \int_2 \text{dr} r^2 = \frac{d \mu_0 I^2}{4 \pi R^4} \frac{R^4}{4} = \frac{1}{2} \mu_0 I^2
\]

\[
L = \frac{d \mu_0}{8 \pi}
\]

\begin{itemize}
  \item \textbf{a)} Direction of current?}
  \begin{center}
    \begin{tikzpicture}
      \draw (-2,0) -- (2,0) -- (2,2) -- (-2,2) -- cycle;
      \draw (-1.5,0) -- (-1.5,2);
      \draw (1.5,0) -- (1.5,2);
      \draw (0,0.5) -- (0,-0.5);
      \draw (-0.5,0.5) -- (-0.5,-0.5);
      \draw (0.5,0.5) -- (0.5,-0.5);
      \draw (1.5,-0.5) -- (1.5,0.5);
      \draw (-1.5,-0.5) -- (-1.5,0.5);
      \node at (0,1) {I};
      \node at (0,-1) {R};
      \node at (-2.5,0) {X};
      \node at (-2.5,2) {X};
      \node at (-0.5,2) {X};
      \node at (0.5,2) {X};
    \end{tikzpicture}
  \end{center}

  \textit{I flows counter-clockwise}

  \Rightarrow B produced by I is out of plane of loop on inside of loop which opposes change in flux as loop falls.

  \item \textbf{b)} \( \varepsilon = -\frac{d}{dt} \Phi \)

  \( \Phi = -Bwy \)

  \( \varepsilon = \frac{d}{dt} Bwy \)

  \( I = \frac{\varepsilon}{R} = \frac{Bwv}{R} \)
c) Calculate forces.

Have \( I \times B \) forces and gravity

\[ F_2 \] and \( F_3 \) cancel since have equal and opposite direction.

\[ F_1 = I \omega B \]
\[ = \frac{B^2 \omega^2 v}{R} \]

Total force in \( y \) direction:

\[ F_y = -mg + \frac{B^2 \omega^2 v}{R} \]

d) Gravity accelerates loop downward. Upward force increases as velocity increases until the two forces balance \( \Rightarrow \) terminal velocity

\[ v_{\text{max}} = \frac{mgR}{B^2 \omega^2} \]
After top of loop enters B

⇒ no more changing flux

⇒ \( E = I = 0 \)

⇒ only gravity so loop will again be accelerated downward

e) energy dissipation rate in resistor

\[
P = I^2 R = \frac{B^2 \omega^2 V_{\text{max}}^2}{R^2}
\]

\[
= \frac{B^2 \omega^2}{R} \cdot \frac{m^2 g^2 R^3}{B^4 \omega^4}
\]

\[
= \frac{m^2 g^2 R}{B^2 \omega^2}
\]

\[
= mg V_{\text{max}}
\]

⇒ energy comes from gravity