Two identical charges with \( q = 5.5 \mu \text{C} \)

have an initial separation \( L = 6.5 \text{ cm} \) and are released with zero initial velocity. Their masses are \( m = 10^{-6} \text{ kg} \). What are their final velocities?

\[ \Delta K = \Delta U = \frac{q^2}{4\pi \varepsilon_0 L} = \frac{1}{2} mv^2 + \frac{1}{2} mu^2 \]

\[ v = \left( \frac{q^2}{4\pi \varepsilon_0 Lm} \right)^{\frac{1}{2}} \]

\[ = 5.5 \times 10^{-6} \text{ m} \]

\[ = 5.5 \times 10^{-6} \times 372 \times 10^9 \text{ m/s} \]

\[ v = 2.0 \times 10^3 \text{ m/s} \]
Consider a small segment $dx$ with charge $dq = dx \frac{Q}{2l}$. The potential at $y$ due to $dq$ is

$$dV = \frac{dq}{4\pi \varepsilon_0 r}$$

$$V = \frac{Q}{8\pi \varepsilon_0 l} \int_{-l}^{l} \frac{dx}{(x^2 + y^2)^{1/2}} = \frac{Q}{4\pi \varepsilon_0 l} \int_{0}^{l} \frac{dx}{(x^2 + y^2)^{1/2}}$$

$$V = \frac{Q}{4\pi \varepsilon_0 l} \ln \left( \frac{l + \left(\frac{y^2}{y^2+\frac{e^2}{y^2}}\right)^{1/2}}{y} \right)$$

Check at large $y$

$$V = \frac{Q}{4\pi \varepsilon_0 l} \ln \left( \frac{l + y \left(1 + \frac{e^2}{y^2}\right)^{1/2}}{y} \right)$$

$$\sim \frac{e}{y}$$

$$V \approx \frac{Q}{4\pi \varepsilon_0 y} \Rightarrow \text{acts like a charge}$$
potential between two parallel plates

\[ V(x) = \frac{8}{m} x + 5.0 \text{ V} \]

\[ E_x = -\frac{\partial}{\partial x} V = -8.0 \frac{\text{V}}{m} \]

At the surface of any conducting plane:

\[ E_n = \frac{E_n}{E_0} \Rightarrow \text{negative charge on left plate and positive on right.} \]

\[ C = \epsilon_0 E_n = -8.85 \times 10^{-12} \text{ C}^2 \quad \frac{8.0}{8.0 \text{ m}^2} \]

\[ = -7.08 \times 10^{-11} \text{ C/m}^2 \]

\[ \Rightarrow \text{same magnitude but positive on the right plate} \]

\[ P 23.49 \]

\[ \begin{align*}
Q_1 & \quad Q_2 \\
\uparrow & \quad \uparrow \\
b & \quad b \\
\downarrow & \quad \downarrow \\
Q_3 & \quad Q_4
\end{align*} \]

\[ U = \frac{1}{4\pi\epsilon_0} \left( \frac{Q_1 Q_2}{b} + \frac{Q_1 Q_3}{b} + \frac{Q_1 Q_4}{b} + \frac{Q_2 Q_3}{12 b} + \frac{Q_2 Q_4}{12 b} + \frac{Q_3 Q_4}{b} \right) \]

All charges are equal to \( Q \).

Add up potential energy of each pair.
\[ U = \frac{1}{4\pi \varepsilon_0} \alpha^2 \left[ \frac{4}{b} + \frac{2}{\sqrt{2}b} \right] \]

\[ U = \frac{\alpha^2}{4\pi \varepsilon_0 b} \left[ 4 + \sqrt{2} \right] \]

b) Add a fifth charge at the center of the square. First calculate the potential at the center.

\[ V = \frac{Q}{4\pi \varepsilon_0 \frac{b}{\sqrt{2}}} + 4 = \frac{\sqrt{2}Q}{\pi \varepsilon_0 b} \]

\[ U = \alpha V = \frac{\sqrt{2} \alpha^2}{\pi \varepsilon_0 b} \]

c) Within plane is it a stable or unstable equilibrium?

\[ \Rightarrow \text{First note that it is in equilibrium since forces sum to zero} \]

\[ \begin{align*}
    \vec{F}_1 &= 0 \\
    \vec{F}_2 &= \frac{Q^2}{4\pi \varepsilon_0 (x-x_0)^2} \\
    \vec{F}_3 &= \frac{Q^2}{4\pi \varepsilon_0 (x+\delta)^2}
\end{align*} \]

To see if unstable, equal. shift particle toward upper right change slightly. How do forces change?

\[ \vec{F}_2 = \frac{Q^2}{4\pi \varepsilon_0 (x-x_0)^2} \]

\[ \vec{F}_3 = \frac{Q^2}{4\pi \varepsilon_0 (x+\delta)^2} \]
$F_2 > F_3$ and $F_2$ is toward center. For small displacements $F_1$ and $F_4$ still balance
\[ \Rightarrow \text{stable equilibrium} \Rightarrow \text{restoring force} \]

d) If $-Q$ at center. Again $F_2 > F_3$ but $F_2$ pulls particle away from center so unstable equilibrium

Electron accelerated through 5500 V.
Calculate velocity from energy cons.
\[ \Delta KE = \Delta U \]
\[ \frac{1}{2}mv_0^2 = eV \]
\[ v_0 = \frac{eV}{me} = \frac{1.6 \times 10^{-19}}{9.11 \times 10^{-31}} \frac{1.1 \times 10^{-4} \text{ Nm}}{\text{J}} \]
\[ = 1.93 \times 10^{15} \frac{\text{m}^2}{\text{s}^2} \]

While in the plates the electron will accelerate upwards. Total time $t_a$ given by
\[ t_a = \frac{L}{v_0} = \frac{6.5 \times 10^{-2} \text{ m}}{4.40 \times 10^7 \text{ m/s}} = 1.48 \times 10^{-9} \text{ s} \]
y velocity:

\[ \frac{dy}{dt} = -eE_y \frac{1}{me} \quad \Rightarrow \quad a_y = -\frac{eE_y}{me} \]

\[ E_y = -\frac{250V}{1.3 \times 10^{-2} \text{m}} = -1.923 \times 10^4 \frac{V}{\text{m}} \]

\[ \Delta v_y = 1.923 \times 10^4 \times \frac{1.6 \times 10^{-19} \text{C}}{9.11 \times 10^{-31} \text{Kg}} \times \frac{1.48 \times 10^{-9} \text{m}}{4 \times 10^{-4} \text{m}} = 5.0 \times 10^6 \frac{\text{m}}{\text{s}} \]

\[ \tan \theta = \frac{\Delta v_y}{v_0} = \frac{5.0 \times 10^6}{4.4 \times 10^7} \]

\[ \theta = 6.5^\circ \]

\[ Q \ 23, \ 15^\circ \]

a) Two conducting spheres one with charge \( Q \) come in contact. The potentials will be the same since they are effectively now a single conductor and the potentials are constant in a conductor.

b) The charge will equally spread between the two spheres—symmetry
c) For unequal radii, the potential difference will be at the same potential, but smaller sphere will have less charge. If each had the same charge the smaller would have a much larger potential. Roughly \( V \approx \frac{Q_{1/2}}{r} \) for the sphere.

(Q 23.17)

The spacing of field lines tells the magnitude of \( E \). Suppose the spacing at a given location is \( \Delta L \) and the potential contours have separation \( \Delta V \). The electric field will roughly be given by (ignoring the sign)

\[ E \approx \frac{\Delta V}{\Delta L}. \]

Thus, smaller spacing means higher \( E \).
a) \( E \) in three regions.

\[
\begin{align*}
\text{if } r > r_2 & \quad \Rightarrow \quad E = \frac{3Q}{8\pi\varepsilon_0 r^2} \\
\text{if } r_1 < r < r_2 & \quad \Rightarrow \quad E = 0 \text{ since inside conductor}
\end{align*}
\]

\[
\begin{align*}
\mathbf{E} & \quad \Rightarrow \quad \mathbf{E} \cdot dA = E \cdot dA = E 4\pi r^2 = \frac{Q}{2 \varepsilon_0} \\
E & = \frac{Q}{8\pi\varepsilon_0 r^2}
\end{align*}
\]

b) Calculate the potential

\[
\begin{align*}
\text{if } r > r_2 & \quad \Rightarrow \quad V(r) - V(r_2) = - \int_{r_2}^{r} E \, dr \\
V(r) & = - \int_{r_2}^{r} \frac{3Q}{8\pi\varepsilon_0 r^2} \, dr = \frac{3Q}{8\pi\varepsilon_0 r} \\
V(r_2) & = \frac{3Q}{8\pi\varepsilon_0 r_2}
\end{align*}
\]

\[
\begin{align*}
\text{if } r_1 < r < r_2 & \quad \Rightarrow \quad V = V(r_2) = \frac{3Q}{Pa\varepsilon_0 r_2} \\
& \text{since } E = 0.
\end{align*}
\]
\[ r < r_1 \]

\[ V(r) - V(r_1) = - \int_{r_1}^{r} \frac{Q}{4\pi \varepsilon_0 r'^2} \, dr' \]

\[ = \frac{Q}{8\pi \varepsilon_0} \left( \frac{1}{r} - \frac{1}{r_1} \right) \]

\[ V(r) = \frac{3Q}{8\pi \varepsilon_0 r_2} + \frac{Q}{8\pi \varepsilon_0} \left( \frac{1}{r} - \frac{1}{r_1} \right) \]