Homework #6 Solutions

P26.18

a) Find the equivalent resistance of

\[ R \quad \frac{R}{R} \quad \frac{R}{3R} \quad \frac{R}{R} \]

Last three add in series \( R' = 3R \)

These add in parallel to give

\[ \frac{1}{R''} = \frac{1}{R} + \frac{1}{R'} = \frac{R + R'}{RR'} \]

\[ R'' = \frac{RR'}{R + R'} = \frac{3R^2}{4R} = \frac{3}{4}R \]

with \( R''' = R'' + \frac{2}{3}R \)

\[ = 2 \frac{3}{4}R \]

\[ \frac{1}{R'''} = \frac{1}{R} + \frac{1}{R''} = \frac{R'''' + R}{RR'''} \]

\[ R'''' = \frac{RR'''}{R + R'''} = \frac{R^2 \cdot 2 \frac{3}{4}}{3 \frac{3}{4}R} = \frac{11}{15}R \]

\[ R_{\text{tot}} = 2R + R'''' = 2 \frac{11}{15}R \]
6) Find current in three resistors if the circuit is connected to a 50 V source.

Know the total current in the circuit.

\[ I = \frac{E}{2 \frac{11}{15} R} = \frac{50 V}{\frac{41}{15} 125 R} = 0.146 A \]

From earlier have

\[ R''' = 2 \frac{3}{4} R = 2.75 R \]

Have \( I = I_1 + I_2 \)

Also know that

\[ I_1 R = I_2 R''' \]

From the loop rule so

\[ I = I_1 + I_1 \frac{R}{R'''} \]

\[ I_1 = \frac{I}{1 + \frac{R}{R''}} = \frac{0.146 A}{1 + \frac{1}{2.75}} = 0.107 A \]

Find currents in each resistor

\[ E_1 = 58 V \]
\[ E_2 = 3 V \]
\[ R_1 = 120 \Omega \]
\[ R_2 = 82 \Omega \]
\[ R_3 = 64 \Omega \]
\[ R_4 = 25 \Omega \]
\[ R_5 = 110 \Omega \]
Use loop rule around two loops:

Left loop: \[ E_1 - I_1 R_1 - I_1 R_2 - I_3 R_3 = 0 \]

Right loop: \[ E_2 - I_2 R_4 + I_3 R_3 - I_2 R_5 = 0 \]

Pot rule: \[ I_2 + I_3 = I_1 \]

Get ride of \( I_3 \) using pot rule

1. \[ E_1 - I_1 R_1 - I_1 R_2 - R_3 (I_1 - I_2) = 0 \]
   \[ E_1 - I_1 (R_1 + R_2 + R_3) + I_2 R_3 = 0 \]

2. \[ E_2 - I_2 R_4 - I_2 R_5 + R_3 (I_1 - I_2) = 0 \]
   \[ E_2 - I_2 (R_4 + R_5 + R_3) + R_3 I_1 = 0 \]

Combining 1 and 2

\[ E_1 - I_1 (R_1 + R_2 + R_3) + R_3 \left[ \frac{E_2 + R_3 I_1}{R_4 + R_5 + R_3} \right] = 0 \]

\[ I_1 = \frac{E_1 + E_2}{R_4 + R_5 + R_3} \frac{R_3}{R_1 + R_2 + R_3 - \frac{R_3^2}{R_4 + R_5 + R_3}} \]
\[ I_1 = \varepsilon_1 + \varepsilon_2 \frac{64}{25 + 110 + 64} \]

\[ = \varepsilon_1 + \varepsilon_2 \frac{64}{199} \]

\[ \times \left( 266 - \frac{64^2}{199} \right) \]

\[ \approx 58 + 3(0.646) \quad A = 0.244 \text{A} \]

Use (1) or (2) to calculate \( I_2 \) and pt. rule to calculate \( I_3 \).

**P 26.50**

Calculate the time constant

\[ \text{a)} \quad \begin{array}{c}
\begin{aligned}
E & \xrightarrow{R_1} I_1 \\
R_2 & \xrightarrow{I_2} \frac{Q}{C} \xrightarrow{-} Q
\end{aligned}
\end{array} \]

\[ \frac{dQ}{dt} = I_c \]

Left loop: \( E - I_1 R_1 - I_2 R_2 = 0 \)

Right loop: \( -\frac{Q}{C} + I_2 R_2 = 0 \quad \Rightarrow I_2 = \frac{Q}{R_2 C} \)

Pt. rule: \( I_1 = I_c + I_2 \)

Want to eliminate \( I_1, I_2 \) so have eqn for \( Q \) and \( I_c = \frac{dQ}{dt} \)

\[ I_1 = \left( E - \frac{Q}{C} \right) \frac{1}{R_1} \]

\[ (E - \frac{Q}{C}) \frac{1}{R_1} = \frac{dQ}{dt} + \frac{1}{R_2 C} Q \]
\[
\frac{dQ}{dt} + Q \left( \frac{1}{R_2C} + \frac{1}{R_1C} \right) = \frac{\varepsilon}{R_1}
\]

Time constant

\[
\frac{1}{\tau} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \frac{1}{C}
\]

\[
\tau = \frac{R_1R_2C}{R_1 + R_2}
\]

b) max. change on cap.?

At late time \(\frac{dQ}{dt} = 0\)

\[
\Rightarrow Q = \frac{\varepsilon C}{R_1} \frac{R_1R_2}{R_1 + R_2}
\]

**P26.90**

When voltage on \(C\) reaches 90 V, it immediately discharges to 65 V and then begins to recharge. Acts like a standard RC circuit until the discharge. From book

\[
V_C = \varepsilon \left( 1 - e^{-t/\tau} \right)
\]

Time to reach 90 V
\[ q_0 = 100(1 - e^{-\frac{t_{90}}{RC}}) \]

\[ e^{\frac{t_{90}}{RC}} = \frac{1}{10} \quad \frac{t_{90}}{RC} = \ln(10) \]

\[ t_{90} = 0.1326 \ln(10) = 0.3045 \text{s} \]

Time to reach 65 V is given by

\[ \frac{65}{100} = 100\left(1 - e^{-\frac{t_{65}}{RC}}\right) \]

\[ e^{\frac{t_{65}}{RC}} = \frac{35}{100} \]

\[ t_{65} = RC \ln(\frac{100}{35}) = 0.139 \text{s} \]

Time to go from 65 V to 90V is

\[ t_{90} - t_{65} = 0.1655 \text{s} \]

Note: takes essentially no time to go back to 65 from 90 since bulb has essentially zero resistance.

P27.12 Calculate force on loop with diverging B

Only downward force survives by symmetry

\[ dF_y = I dB \sin \theta \]
\[ F_y = IBr \sin \theta \quad Sd \ell = IBr \sin \theta \cdot 2\pi r \]

\[ \tan \theta = \frac{r}{d}, \quad \sin \theta = \frac{r}{\sqrt{r^2 + d^2}} \]

\[ F_y = IB \frac{2\pi r^2}{(r^2 + d^2)^{\frac{3}{2}}} \]

---

**P 27.16**  
**Force on a negative charge**

\[ \mathbf{F}_n = q \mathbf{n} \times \mathbf{B} \]

\[ \text{a)} \quad \mathbf{F}_n \quad \text{b)} \quad \mathbf{F}_n \quad \text{c)} \]

\[ \text{d)} \quad \mathbf{B} \quad \mathbf{n} \quad \mathbf{E}_n \quad \text{e)} \quad \mathbf{F} \quad \mathbf{n} = 0 \]

---

**Q 26.6**  
Given two light bulbs and two batteries. Arrange to get max power to bulbs.

Each bulb has a potential of 2\( \mathcal{E} \).
**Q 27.3**

\[ F = I \frac{\ell}{m} \times B \]

\( \ell \) is in along \( I \) and \( B \) is to the right.

**Force is down.**

**P 27.11**

Show that the force is the same as that on a straight wire from \( a \) to \( b \).

Choose coordinates with \( x \) from \( a \) to \( b \) and \( y \perp \) to the line from \( a \) to \( b \).

\[ dF = I \frac{dx}{m} \times B \]

Let \( dx = dx \hat{\mathbf{i}} + dy \hat{\mathbf{j}} \)

\[ dF = I \left( dx \hat{\mathbf{i}} + dy \hat{\mathbf{j}} \right) \times B(-k) \]

\[ = I B \left( - \hat{\mathbf{k}} \times \hat{\mathbf{i}} dx - \hat{\mathbf{k}} \times \hat{\mathbf{j}} dy \right) \]

\[ = I B \left( dx \hat{\mathbf{j}} - dy \hat{\mathbf{i}} \right) \]

Integrate from \( a \) to \( b \).

\[ F = I B \int_{a}^{b} \left( \int dy - \int dx \right) \]

\[ F = I B \int_{a}^{b} \left( b - a \right) \]

same as straight wire.