Calculate magnetic flux through the loop.

\[ B = \frac{\mu_0 I}{2\pi r} \]

\[ \Rightarrow B \text{ varies with } r \text{ so consider the slice shown.} \]

Flux through slice \( d\Phi = B d\alpha \)

\[ d\Phi = \frac{\mu_0 I a}{2\pi r} \frac{a}{b} dr \]

\[ \Phi = \frac{\mu_0 I a}{2\pi} \ln \left( \frac{b + a}{b} \right) \]

For path around loop shown:

\[ E = - \frac{\mu_0 a}{2\pi} \ln \left( \frac{b + a}{b} \right) \frac{dI}{dt} \]

\[ = - \frac{2 \times 10^{-7} \, T \cdot m}{A} \times 0.12 \, m \ln \left( \frac{22}{15} \right) \frac{2500 \cos(2500t)}{s} \]

\[ = - 2.4 \times (1.5) \times 10^{-4} \frac{A}{m} \ln \left( \frac{22}{15} \right) \times \frac{V}{A} \times \frac{2500 \cos(2500t)}{s} \]

\[ E = - 3.5 \times 10^{-5} \cos(2500t) \, V \]
\[ V(t) = 0 \text{ at } t = 0 \]
\[ F = I B L \times B \]
\[ \Rightarrow \text{ to right} \]
\[ F = I L B \]

a) For constant \( I \), \( F \) is a constant so
\[ a = \frac{I L B}{m} \]
\[ V(t) = \frac{I L B}{m} t \]

b) For constant \( E \), there will be a back emf \( E_b \) that will oppose the increased magnetic flux through the loop. This will oppose \( E \) and therefore reduce the current.
\[ \Delta E = - \frac{d}{dt} (BA) = -B \frac{dA}{dt} \]
\[ = -B L V \]
\[ E + \Delta E - IR = 0 \Rightarrow \text{loop rule} \]
\[ I = \frac{E - B L V}{R} \]

\[ m \frac{dv}{dt} = F = L B \left( \frac{E - B L V}{R} \right) \Rightarrow \text{terminal speed} \]
\[ \frac{dv}{dt} + \frac{B^2 L^2}{m R} V = \frac{L B}{R m} E \]
\[ \text{characteristic time:} \quad \frac{m R}{B^2 L^2} \]
\[
\frac{dv}{dt} + \frac{v}{\gamma} = \frac{v_T}{\gamma}
\]

\[
v = \frac{v_T - v}{\gamma} \frac{d}{dt}
\]

\[
\int_0^v \frac{dv}{v_T - v} = \int_0^t \frac{dt}{\gamma} = \frac{t}{\gamma}
\]

\[- \ln (v_T - v) \bigg|^v_0 = \frac{t}{\gamma}
\]

\[
\frac{v_T - v}{v_T} = e^{-\frac{t}{\gamma}}
\]

\[
v(t) = v_T \left( 1 - e^{-\frac{t}{\gamma}} \right)
\]

(c) terminal speed: \( v_T = \frac{e}{B} \)

P29, 40

Circular coil rotates at 120 rev/s = \( \omega \) and has 250 loops, in a \( B = 0.45 \) T field. RMS output? \( R = 5 \) cm

\[
E = -\frac{d}{dt} \int_B B \cdot d\mathbf{A} = -\frac{d}{dt} B R A \cos \theta
\]

\[
\theta = \omega t = 2\pi f t
\]

\[
E = +BR^2 A R^2 \sin(\omega t) 2\pi f
\]

\[
E_{me} = \left< E^2 \right> = B^2 R^2 \frac{1}{2\pi} \left< \sin^2 \omega t \right> = \frac{BR^2 E^2}{12}
\]
\[ E_{rms} = \frac{0.45}{1.2} \frac{T}{\text{m}^2} \frac{(20)^2}{5} \frac{2\pi^2}{12} \]

\[ I_T = \frac{N}{Am} \]

\[ \frac{Tm^2}{s} = \frac{N m^2}{Am^2} = \frac{T}{Q} = \mathcal{V} \]

\[ E_{rms} = 0.6 \frac{1.2}{6} \frac{2\pi^2}{12} \mathcal{A} = 1.88 \mathcal{V} \]

\[ P = 29.64 \]

\[ \text{Change at } x = 0.1m \]

\[ Q = 0.1 \mu C \]

\[ B \text{ decreases with } \]

\[ \frac{dB}{dt} = -0.1 T/s \]

Electric field lines are circles in clockwise direction.

\[ \oint E \cdot dl = E 2\pi r = -\pi r^2 \frac{dB}{dt} \]

\[ E = -\frac{r}{2} \frac{dB}{dt} \]

\[ F = -Q \frac{r}{2} \frac{dB}{dt} \]

\[ \frac{GTm}{s} = \frac{Q N m}{Am^2} = N \]

\[ F = 10^{-9} N \]
The orbiting electrons maintain their circular orbit by an inward radial force

\[ F = -e \frac{ve \times B}{m} \]

The changing \( B \) produces a circular \( E \) that is clockwise to accelerate the clockwise circling electrons.

b) Electrons move clockwise so \( E \) is radially inward counterclockwise.

c) Want \( E \) clockwise. If \( B \) increases, \( E \) will be counterclockwise \( \Rightarrow \) Lenz's law.

d) \( B \)

Take \( B \) positive inward. Want \( B \) positive and increasing.

\( \Rightarrow \) only first quarter of cycle.
Consider electrons on a thin strip as shown. They move with velocity $v = \omega r$ across $B$

They will experience an outward force $F = e\omega r B$

Total work done from $r = 0$ to $r = R$

$$\mathcal{W} = e\omega B \int_0^R r \, dr = e\omega B \frac{R^2}{2}$$

This is equal to $\mathcal{E}$

$$\Rightarrow \mathcal{E} = \frac{e\omega R^2}{2} B$$

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**Problem 29.76**

Cylindrical inductor $N = 2000$ turns

$R = 1.25 \text{ cm}$

$L = 21.7 \text{ cm}$

$E \times 10^{-4}$

$$L = \frac{\mu_0 N^2 A}{L} = 4\pi \times 10^{-7} \frac{Tm^2}{A} \frac{(2.8)^2}{10 \pi (1.25)^2 m^2} \cdot 21.7 m$$

$$\frac{Tm^2}{A} = \frac{N}{Am} \frac{m^2}{A} = \frac{J}{A^2} = H$$

$$L = 4\pi^2 (2.8)^2 (1.25)^2 \times 10^{-5} H = 2.2 \times 10^{-2} H$$
A toroid with a rectangular cross section. What is the self-inductance?

For the current shown, \( B \) will be counted clockwise. Use Ampere's law along path shown:

\[
\oint B \cdot dl = B \cdot 2\pi v = I_{\text{tor}} M_0
\]

\[
B = \frac{N I M_0}{2\pi v}
\]

\[
L = \frac{N M_0}{I}
\]

\[
\Phi = \oint B \cdot dl = \frac{N I M_0 h}{2\pi v}
\]

\[
\Phi = \frac{N I M_0 h}{2\pi} \int_{v_1}^{v_2} \frac{dv}{v} = \frac{N I M_0 h}{2\pi} \ln \left( \frac{v_2}{v_1} \right)
\]

\[
L = \frac{N^2 I^2 M_0 h}{2\pi} \ln \left( \frac{v_2}{v_1} \right)
\]