Physics 604 Midterm #1 Fall '11

Dr. Drake

1. (a) (20 points) Evaluate

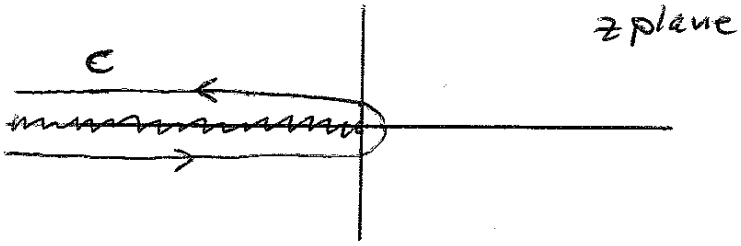
$$\int_{-\infty}^{\infty} dz \frac{\sin^2 z}{z^2}$$

Hint: $\sin^2 z = (1 - \cos(2z))/2$

(b) (20 points) Evaluate

$$\int_C dz \frac{1}{z^{3/2}(z^2 + a^2)}$$

with the contour C given by

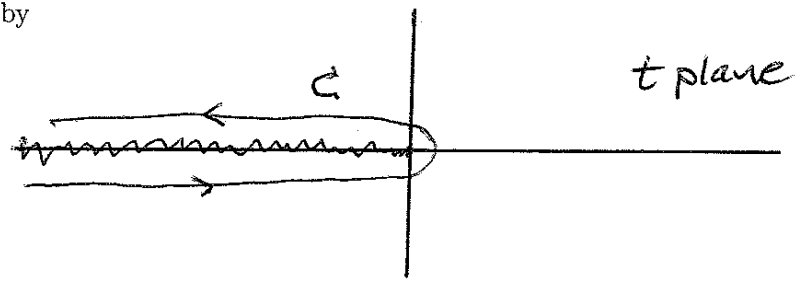


with a real and positive. The contour lies very close to the cut.

2. (30 points) Consider the following integral

$$Q_\nu(z) = \int_C dt e^{zt} t^\nu$$

with $-\pi < Arg(t) < \pi$, where ν is a real, positive number and the contour C given by



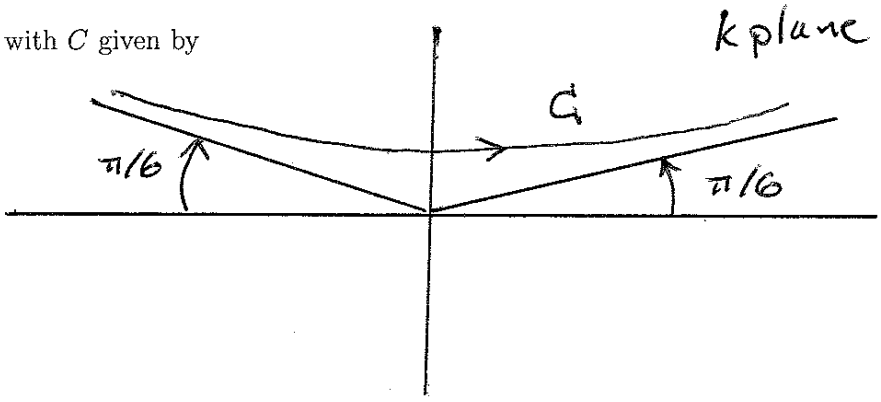
10 (a) For what values of complex z is the integral defined?

20 (b) Evaluate $Q_\nu(re^{-i\pi})$ with r real and positive by analytic continuation. Express your answer in terms of $Q_\nu(r)$.

3. (30 points) An integral representation for the Airy function is given by

$$A_i(z) = \int_C dk e^{ikz} e^{ik^3/3}$$

with C given by



Take z to be large, real and positive.

10 (a) Indicate where the integrand is large and small in the k plane.

10 (b) Find the location of the saddle points and indicate the directions of the PSDs.

10 (c) Evaluate $A_i(z)$.