1. (a) (20 points) Evaluate

\[ \int_{-\infty}^{\infty} dz \frac{\sin^2 z}{z^2} \]

Hint: \( \sin^2 z = (1 - \cos(2z))/2 \)

(b) (20 points) Evaluate

\[ \int_{C} dz \frac{1}{z^{3/2}(z^2 + a^2)} \]

with the contour \( C \) given by

with \( a \) real and positive. The contour lies very close to the cut.

2. (30 points) Consider the following integral

\[ Q_\nu(z) = \int_{C} dt e^{zt} t^\nu \]

with \(-\pi < \text{Arg}(t) < \pi\), where \( \nu \) is a real, positive number and the contour \( C \) given by

10 (a) For what values of complex \( z \) is the integral defined?

20 (b) Evaluate \( Q_\nu(re^{-it}) \) with \( r \) real and positive by analytic continuation. Express your answer in terms of \( Q_\nu(r) \).
3. (30 points) An integral representation for the Airy function is given by

\[ A_i(z) = \int_C \kappa e^{ik\kappa} e^{i\kappa^3/3} \]

with \( C \) given by

Take \( z \) to be large, real and positive.

(i) (a) Indicate where the integrand is large and small in the \( k \) plane.

(ii) (b) Find the location of the saddle points and indicate the directions of the PSDs.

(iii) (c) Evaluate \( A_i(z) \).