Physics 604 | Homework #5 | Fall '20
Dr. Drake

Note that the Arfken problems correspond to the 7th Edition with the corresponding 6th Edition problems in the parentheses.

1. Arfken 14.6.1 (7.3.4), 14.6.2 (7.3.5)
2. Evaluate the following integral

$$\int_0^1 dte^{ixt^2},$$

   for large x with x real and positive. You will find that the lowest order term scales as $x^{-1/2}$ and the next order as e^{ix}/x. Calculate both terms.

   Hint: The real axis is not the best contour of integration.

3. Evaluate the following integral for x large and real

$$\int_0^\pi dte^{ix\cos t}.$$

4. Consider the following integral

$$I(z) = \int_0^\infty dte^{-zt} ln(t)$$

   with $-\pi < Arg(t) < \pi$.

   (a) For what values of complex z is the integral defined?
   (b) Evaluate $I(re^{-i\pi})$ with r real and positive by analytic continuation. Express your answer in terms of $I(r)$.

5. (challenge problem: not for grading) Find the analytic continuation of $K_\nu(z)$:

$$K_\nu(re^{i\pi}) = e^{-i\nu\pi}K_\nu(r) - i\pi I_\nu(r)$$

   with r real and positive and

$$K_\nu(z) = \frac{1}{2}\int_0^\infty \frac{e^{-\frac{z}{2}(s+\frac{1}{s})}}{s^{1+\nu}}.$$

   Hint: Let $z = re^{i\theta}$ and let θ increase from 0 to π. As you do this make sure that the integral remains bounded both around $s = 0$ and ∞.