

1. The Coulomb Wave equation is given by

$$\frac{d^2w}{dz^2} + \left[1 - \frac{2\eta}{z} - \frac{L(L+1)}{z^2} \right] w = 0$$

- (a) Classify the singular points of this equation.
- (b) Find the behavior of the solutions for z very close to zero. Show that the solutions are linearly independent. Are there special cases where they are not linearly independent? What are the solutions in this case?
2. The Legendre equation is given by

$$(1 - z^2) \frac{d^2w}{dz^2} - 2z \frac{dw}{dz} + \alpha(\alpha + 1)w = 0.$$

- (a) Classify the singular points of this equation.
- (b) Find the behavior of the solutions for z very close to 1. Show that the solutions are linearly independent.
- (c) Calculate the full series solution around $z = 1$.
Note: This is a degenerate case. You only need to calculate the solution that is non-singular around $z = 1$. Obtaining the second full series solution is messy.
3. Bessel's equation is given by

$$z^2 \frac{d^2w}{dz^2} + z \frac{dw}{dz} + (z^2 - \nu^2)w = 0.$$

- (a) Identify the location of the singular points of this equation.
- (b) Find the lowest order behavior of the solution around $z = 0$. What is the behavior if $\nu = 0$.