

Note that the Arfken problems correspond to the 7th Edition with the corresponding 6th Edition problems in the parentheses.

1. Arfken 7.6.12 (9.6.11)
2. Bessel's equation is given by

$$z^2 \frac{d^2 y}{dz^2} + z \frac{dy}{dz} + (z^2 - \nu^2)y = 0.$$

Convert this equation to the standard form

$$\frac{d^2 y}{dz^2} + k^2(z)y = 0$$

and calculate the WKB solutions for large $|z|$. Where are the solutions valid? Compare your results to the asymptotic form of the Hankel function obtained from the integral representation (in the book).

Hint: When carrying out the integral over the phase, use the fact that $|z|$ is large to simplify the integral. The final answer should take a very simple form.

3. The parabolic cylinder equation is given by

$$\frac{d^2 y}{dz^2} - \left(\frac{1}{4}z^2 + a\right)y = 0.$$

Repeat the above for this equation. The hint from the previous problem also applies here.