Maxwell's Equations

\[ \nabla \times \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} \]
\[ \mathbf{D} = \epsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P} \]
\[ \nabla \cdot \mathbf{E} = 0 \]
\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 \mathbf{H} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \mathbf{B} = \nabla \times \mathbf{A} \]
\[ \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{A} - \nabla \Phi \]

Math eqns

\[ \nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \]
\[ \nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial \theta^2} \]

\[ \nabla^2 G(r,r') = -4\pi \delta(r-r') \]
\[ G(r,r') = \frac{1}{r-r'} \]

Bessel Eqn:

\[ \frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} + \left(1 - \frac{\nu^2}{x^2}\right) = 0 \]

\[ J_\nu(x) \sim x^\nu \text{(small x)} \sim \frac{1}{x^{\nu+1}} \cos \left(x - \frac{\pi \nu}{2} - \frac{\pi}{4}\right) \text{(large x)} \]
\[ Y_\nu(x) \sim x^{-\nu} \text{(small x)} \sim \frac{1}{x^{\nu+1}} \sin \left(x - \frac{\pi \nu}{2} - \frac{\pi}{4}\right) \text{(large x)} \]
\[ J_{-\nu}(x) \sim Y_{\nu}(x) \]
\[ N_{\nu}(x) \sim Y_{\nu}(x) \]

\[ H_\nu^{(1)}(x) = J_\nu + i N_\nu \]
\[ H_\nu^{(2)}(x) = J_\nu - i N_\nu \]

Modified Bessel Eqn:

\[ \frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} - \left(1 + \frac{\nu^2}{x^2}\right) = 0 \]

\[ I_\nu(x) \sim x^\nu \text{(small x)} \sim \frac{1}{x^{\nu+1}} e^x \text{(large x)} \]
\[ K_\nu(x) \sim x^{-\nu} \text{(small x)} \sim \frac{1}{x^{\nu+1}} e^{-x} \text{(large x)} \]
\[ a \int_0^\infty r J_\nu \left(\sqrt{\frac{a^2}{2} + r^2}\right) \, dr = a \frac{1}{2} J_{\nu+1}(ax) \]

Legendre Eqn:

\[ \frac{d}{dx} (1-x^2) \frac{d}{dx} + (\ell+1) - \frac{m^2}{1-x^2} = 0 \]
\[ \sum |Y_{lm}|^2 = 1 \quad \text{for } Y_{lm} = \sqrt{\frac{2l+1}{4\pi} \frac{(2m)!}{(2l-m)!}} \cdot \text{Re}(c_0) \cdot e^{i\mu \theta} \]

**Waves**

\[ \Delta G = D - \frac{1}{c^2} \partial_t \partial_x \Delta x \]

\[ G = \frac{1}{l^2} \delta(l - (c - l' \partial_x) \frac{1}{l}) \]

wave eqn: \[ \nabla^2 G + \frac{\omega^2}{c^2} \left( \frac{1}{l} \right) = 0 \]

**Electrostatics**

- charge: \[ \mathcal{E}(x) = \frac{\mathcal{E}}{4\pi \varepsilon_0} \delta(x) \]
- dipole: \[ \mathcal{E}(x) = \frac{1}{4\pi \varepsilon_0} \frac{P \cdot x}{|x|^3} \]

magneto statics

\[ dB = \nabla \times \mathbf{B} \]

\[ d\mathbf{F} = \mathbf{I} \times d\mathbf{x} \]

\[ \mathbf{E} = \frac{\mathcal{E}}{4\pi \varepsilon_0} \]

\[ \mathbf{E} = \mathbf{E}_0 e^{i\mathbf{k} \cdot \mathbf{x}} \]

\[ \mathbf{B} = \mathbf{B}_0 e^{i\mathbf{k} \cdot \mathbf{x}} \]

\[ \mathbf{U}_E = \frac{1}{2} \mathbf{E} \cdot \mathbf{E} \]

\[ \mathbf{U}_B = \mathbf{B} \cdot \mathbf{B} \]

\[ \mathbf{U}_p = \mathbf{P} \cdot \mathbf{E} \]

\[ \mathbf{U}_m = \mathbf{M} \times \mathbf{E} \]

\[ \mathbf{U}_m = \frac{1}{2} \mathbf{B} \times \mathbf{B} \]

\[ \mathbf{B} = \nabla \times \mathbf{B} \]

spec rel

\[ \beta = \frac{v}{c} \quad \gamma = 1 / (1 - \beta^2)^{1/2} \]

\[ \left( \begin{array}{c} x_0' \\ x_1' \\ x_2' \\ x_3' \end{array} \right) = \left( \begin{array}{cccc} \gamma & -\beta \gamma & 0 & 0 \\ -\beta \gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \left( \begin{array}{c} x_0 \\ x_1 \\ x_2 \\ x_3 \end{array} \right) \]

\[ \mathbf{x} = \gamma m c^2 \quad \mathbf{p} = \gamma m \mathbf{v} \]

velocity:

\[ u^x = \frac{u^x + v}{1 + \frac{v u^x}{c^2}} \quad u^\perp = \frac{u^\perp}{\sqrt{1 + \frac{v^2 u^\perp}{c^2}}} \]

fields:

\[ E_{\perp} = E_0 \quad B_{\perp} = B_0 \]

\[ \frac{1}{\gamma} E_\parallel = \gamma (\frac{1}{\gamma} E_\parallel + \gamma^2 \mathbf{B} \times \mathbf{B}) \quad B_\parallel = \frac{1}{\gamma} (B_\perp - \gamma \mathbf{B} \times E_\perp) \]