MATH 808G (Fall 2018) - Introduction to categorifications of quantum groups

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"Categorification" from Wikipedia: In mathematics, categorification is the process of replacing set-theoretic theorems by category-theoretic analogues. Categorification, when done successfully, replaces sets by categories, functions with functors, and equations by natural isomorphisms of functors satisfying additional properties. The term was coined by Louis Crane.

Course description: The goal of this course is to introduce the idea of categorification in representation theory, which has become a powerful tool and an active area of research.

We shall only focus on the categorification of the quantum group $\mathbf{U}_q(\mathfrak{sl}_2)$ in this course. We shall start by introducing the quantum group $\mathbf{U}_q(\mathfrak{sl}_2)$ and then study its (diagrammatic) categorification. We shall follow Lauda's paper for the first part of this course, with supplementary materials from Lusztig's book and Rouquier's paper. In the second part of the course, we study the application following the paper by Chuang and Rouquier. We shall decide how we cover the materials for this part depending on the interests of the people attending.

References: We do not have a textbook for this course. But the following are the main references we shall use:

- A. Lauda, *A categorification of quantum* $\mathfrak{sl}(2)$. Adv. Math. **225** (2010), 3327–3424.
- J. Chuang and R. Rouquier, *Derived equivalences for symmetric groups and* $\mathfrak{sl}_2$*-categorification*, Ann. of Math. (2) **167** (2008), 245–298.

Additional references:

- J.C. Jantzen, *Lectures on Quantum Groups*, Graduate Studies in Mathematics **6**, Amer. Math. Soc, 1996.
- A. Lauda, *An introduction to diagrammatic algebra and categorified quantum* $\mathfrak{sl}_2$, Bulletin of the Institute of Mathematics, Academia Sinica (Ner Series) **7** (2012), No. 2, pp. 165-270.
- A. Lauda, *Categorified quantum* $\mathfrak{sl}(2)$ *and equivariant cohomology of iterated flag varieties*, Algebras and Representation Theory, **14** (2011), 253–282.
- G. Lusztig, *Introduction to Quantum Groups*, Modern Birkhäuser Classics, Reprint of the 1993 Edition, Birkhäuser, Boston, 2010.
- R. Rouquier, *2-Kac-Moody algebras*, arXiv:0812.5023.
- M. Khovanov and A. Lauda, *A diagrammatic approach to categorication of quantum groups I & II & III*, Represent. Theory **13** (2009), 309-347; Quantum Topology **1** (2010), 1-92; Trans. Amer. Math. Soc. **363** (2011), 2685-2700.

Prerequisites: I will assume you have taken Algebra I&II and familiar with basic category theory (functors, natural transformations, etc.). I will also assume you are familiar with the Lie algebra $\mathfrak{sl}_2(\mathbb{C})$ (and its representation theory). That is all you need to know for the first part.

The second part requires more advanced background (e.g. derived categories). But I will keep it minimal. We shall also decide how we cover the materials for this part depending on the interests of the people attending.

For those taking this course for a grade: I hope that you will attend the lectures and ask questions. In addition, I may assign a few homework problems (to verify some statements made in class or to work out some specific examples). There will be no exams in this course.

Homework: There will be several homework assignments.