%Rosemond Boateng Extra Credit

%problem 5 on 9.4

syms x y
sys1 = x*(1-x-y);
sys2 = y*(1.5-y-x);
[xc, yc] = solve(sys1, sys2, x, y);
disp ('Critical Points:'); disp ([xc yc])

warning off all
f1 = @(t, x) [x(1)*(1-x(1)-x(2)); x(2)*(1.5-x(2)-x(1))];
figure; hold on
for a = -5: 0.5:2.5
    for b = 0.25: 0.25:2.5
        [t, xa] = ode45(f1, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f1, [0 -10], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end

title 'Problem 5'

axis ([0 2 0 2])

[X Y] = meshgrid (0:0.25:2, 0:0.25:2);
F1 = X.*(1-X-Y);
F2 = Y.*(1.5-Y-X);
L = sqrt((F1/3).^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);
hold off

% The critical points are:
% (0,0) is a nodal source.
% (0, 1.5) is a nodal sink
% (1,0) is a saddle point.
% This is a rabbit-squirrel competition relationship. Neither species likes
% seeing the other species nor themselves because they are competing for
% the same food supply. At (0,0), the population of both species is
% increasing and approaching (1.5,0). If either species gets above the line
% y= -1.5*x +1.5, the population of both species will decrease, then try to
% increase and approach (1.5,0).

Critical Points:
[ 0, 0]
[ 0, 3/2]
[ 1, 0]
problem 6

```matlab
syms x y
sys1 = x*(1 - x + 0.5*y);
sys2 = y*(2.5 - 1.5*y + 0.25*x);
[xc, yc] = solve(sys1, sys2, x, y);
derm ('Critical Points:'); disp ([xc yc])

warning off all

f2 = @(t, x) [x(1)* (1 - x(1) + 0.5*x(2)); x(2)* (2.5 - 1.5*x(2) + 0.25*x(1))];
figure; hold on
```

Problem 5
for a = 0.25: 0.75:2.5
    for b = 0.25: 0.25:2.5
        [t, xa] = ode45(f2, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f2, [0 -10], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end

title 'Problem 6'
axis ([0 4 0 4])

[X Y] = meshgrid (0:0.25:4, 0:0.25:4);
F1 = X.*(1- X + 0.5.*Y);
F2 = Y.*(2.5 - 1.5.*Y + 0.25.*X);
L= sqrt((F1/3).^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);
hold off

% (0,0) is a nodal source.
% (0,5/3) is a saddle
% (1,0) is a also a saddle
% (2,2) is a nodal sink

Critical Points:
[    0,    0]
[    0, 5/3]
combining prob 5 and 6 with alpha =c=0

warning off all

c = 0

syms x y

sys1 = (((1-c)*(x*(1 - x - y)))+ ((c)*(x*(1 - x + 0.5*y))));

sys2 = (((1-c)*(y*(1.5 - y - x)))+((c)*(y*(2.5 - 1.5*y + 0.25*x))));

[xc, yc] = solve(sys1, sys2, x, y);

disp ('Critical Points:'); disp ([xc yc])
f1 = @(t, x) [x(1)* (1 - x(1) - x(2)); x(2)* (1.5 - x(2) - x(1))];

f2 = @(t, x) [x(1)* (1 - x(1) + 0.5*x(2)); x(2)* (2.5 - 1.5*x(2) + 0.25*x(1))];

f3 = @(t, x) [((1-c)*(x(1)^2) - x(1) - x(2)) + (c*(x(1)^2) - x(1) + 0.5*x(2)));

figure; hold on
for a = -5: 0.5:2.5
    for b = 0.25: 0.25:2.5
        [t, xa] = ode45(f3, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f3, [0 -10], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
end

title 'Problem 5 and 6 with c= 0'
axis ([0 2 0 2])
[X Y] = meshgrid (0:0.25:2, 0:0.25:2);
F1 = (((1-c)*X.* (1 - X - Y)) + (c.*X.* (1 - X + 0.5.*Y)));
F2 = (((1-c).*Y.* (1.5 - Y - X)) + (c.*Y.* (2.5 - 1.5.*Y + 0.25.*X)));
L= sqrt((F1/3).^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);
hold off

% The critical points are:
% (0,0) is a nodal source.
% (0, 1.5) is a nodal sink
% (1,0) is a saddle point.

% This is a rabbit-squirrel competition relationship. Nither species likes
% seeing the other species nor themselves because they are competing for
% the same food supply. At (0,0), the population of both species is
% increasing and approaching (1.5,0). If ei
% ther species gets above the line
% y= -1.5*x +1.5, the population of both species will decrease, then try to
% increase and approach (1.5,0).

c =

    0

Critical Points:
[ 0, 0]
[ 0, 3/2]
[ 1, 0]
combining prob 5 and 6 with alpha =c=1/6

warning off all

c = 1/6

syms x y

sys1 = (((1-c)*(x*(1-x-y)))+((c)*(x*(1-x +0.5*y))));

sys2 = (((1-c)*(y*(1.5-y-x)))+((c)*(y*(2.5-1.5*y +0.25*x))));

[xc, yc] = solve(sys1, sys2, x, y);

disp ('Critical Points:'); disp ([xc yc])

f1 = @(t, x) [x(1)* (1 - x(1) - x(2)); x(2)* (1.5 - x(2) - x(1))];
\( f_2 = @ (t, x) [x(1)* (1 - x(1) + 0.5*x(2)); x(2)* (2.5 - 1.5*x(2) + 0.25*x(1))]; \)

\( f_3 = @ (t, x) [(((1-c)*(x(1)* (1 - x(1) - x(2))))+ (c*(x(1)* (1 - x(1) + 0.5*x(2))))); (((1-c)*(x(2)* (1.5 - x(2) - x(1))))+((c)*(x(2)* (2.5 - 1.5*x(2) + 0.25*x(1)))))); \)

figure; hold on

for a = -0.25: 0.75:2
    for b = -0.25: 0.25:2
        [t, xa] = ode45(f3, [-5 10], [a b]);
        plot(xa(:,1), xa(:,2))
    [t, xa] = ode45(f3, [-5 -10], [a b]);
    plot(xa(:,1), xa(:,2))
    end
end

title 'Problem 5 and 6 with c= 1/6'

axis([-0.5 2 0 2])

[X Y] = meshgrid (-0.5:0.25:2, 0:0.25:2);

F1 = (((1-c).*(X.* (1 - X - Y)))+ ((c).*((X.* (1 - X + 0.5.*Y))));

F2 = (((1-c).*(Y.* (1.5 - Y - X)))+((c).*((Y.* (2.5 - 1.5.*Y + 0.25.*X))));

L= sqrt((F1/3).^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);

hold off

% The critical points are:
% (0,0) is a nodal source.
% (0, 20/13) is a nodal sink
% (1,0) is also a saddle
% (-16/47, 84/47) is also saddle
% It's still a rabbit-squirrel competition relationship but both species
% increase from (0,0) until they get close to the curve going from 1.54 to
% approximately 1, then they approach the point (0, 1.54). One of the
% species has started to evolve causing the line when c=0 to change into a
% curve.

c =

0.1667

Critical Points:
[ 0, 0]
[ 0, 20/13]
[ 1, 0]
[ -16/47, 84/47]
combining prob 5 and 6 with alpha =c=1/3

warning off all

c = 1/3

syms x y

sys1 = (((1-c)*(x*(1-x-y)))+((c)*(x*(1-x+y))));
sys2 = (((1-c)*(y*(1.5-y-x)))+((c)*(y*(2.5-1.5y+0.25x))));

[xc, yc] = solve(sys1, sys2, x, y);
disp ('Critical Points:'); disp ([xc yc])

f1 = @(t, x) [x(1)* (1 - x(1) - x(2)); x(2)* (1.5 - x(2) - x(1))];
\[ f_2 = \mathbb{R}(t, x) \left[x(1) \cdot (1 - x(1) + 0.5 \cdot x(2)) \cdot x(2) \cdot (2.5 - 1.5 \cdot x(2) + 0.25 \cdot x(1))\right]; \]

\[ f_3 = \mathbb{R}(t, x) \left[(((1 - c) \cdot (x(1) \cdot (1 - x(1) - x(2)))) + (c \cdot (x(1) \cdot (1 - x(1) + 0.5 \cdot x(2))))); (((1 - c) \cdot (x(2) \cdot (1.5 - x(2) - x(1))))) + ((c) \cdot (x(2) \cdot (2.5 - 1.5 \cdot x(2) + 0.25 \cdot x(1))))\right]; \]

figure; hold on

for \( a = 0.25 \): 0.75:2.5
  for \( b = 0.25 \): 0.25:3
    \[ [t, xa] = \text{ode45}(f_3, [0 \ 10], [a \ b]); \]
    \[ \text{plot}(xa(:,1), xa(:,2)) \]
    \[ [t, xa] = \text{ode45}(f_3, [0 \ -10], [a \ b]); \]
    \[ \text{plot}(xa(:,1), xa(:,2)) \]
  end
end

title 'Problem 5 and 6 with c= 1/3'

axis ([0 2 0 2])

\[ [X \ Y] = \text{meshgrid}(0:0.1:2, 0:0.25:2); \]

\[ F_1 = (((1-c) \cdot (X \cdot (1 - X - Y))) + (c) \cdot (X \cdot (1 - X + 0.5 \cdot Y))); \]

\[ F_2 = (((1-c) \cdot (Y \cdot (1.5 - Y - X))) + (c) \cdot (Y \cdot (2.5 - 1.5 \cdot Y + 0.25 \cdot X))); \]

\[ L = \sqrt{(F_1/3)^2 + (F_2/6)^2}; \]

\[ \text{quiver}(X, Y, F_1/L, F_2/L, 0.5); \]

hold off

\% The critical points are:

\% (0,0) is a nodal source

\% (0,11/7) is a saddle
% (1,0) is also a saddle
% (2/7, 10/7) is a nodal sink
% It's still a rabbit-squirrel competition relationship. One of the species
% has evolved some more causing the curve to widen at the base. Both
% species increase from (0,0) and approach (2/7, 10/7). If any of the
% species get above the curve, their population will decrease then increase
% and approach (2/7, 10/7).

c =

0.3333

Critical Points:
[ 0, 0]
[ 0, 11/7]
[ 1, 0]
[ 2/7, 10/7]
combining prob 5 and 6 with alpha = c = 1/2

warning off all

c = 1/2

syms x y

sys1 = (((1-c)*(x*(1-x-y)))+((c)*(x*(1-x+0.5*y))));

sys2 = (((1-c)*(y*(1.5-y-x)))+((c)*(y*(2.5-1.5*y+0.25*x))));

[xc, yc] = solve(sys1, sys2, x, y);

disp ('Critical Points:') ; disp ([xc yc])

f1 = @(t, x) [x(1)* (1 - x(1) - x(2)); x(2)* (1.5 - x(2) - x(1))];
f2 = @(t, x) [x(1)* (1 - x(1) + 0.5*x(2)); x(2)* (2.5 - 1.5*x(2) + 0.25*x(1))];
f3 = @(t, x) [(((1-c)*(x(1)* (1 - x(1) - x(2))))+ (c*(x(1)* (1 - x(1) + 0.5*x(2))))); (((1-c)*(x(2)* (1.5 - x(2) - x(1))))+(c)*(x(2)* (2.5 - 1.5*x(2) + 0.25*x(1))))];

figure; hold on

for a = 0.25: 0.75:3
    for b = 0.25: 0.25:3
        [t, xa] = ode45(f3, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))

        [t, xa] = ode45(f3, [0 -10], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end

title 'Problem 5 and 6 with c= 1/2'
axis ([0 2 0 2])

[X Y] = meshgrid (0:0.2:2, 0:0.25:2);
F1 = (((1-c).*(X.* (1 - X - Y)))+ (c).*(X.* (1 - X + 0.5.*Y)));
F2 = (((1-c).*(Y.* (1.5 - Y - X)))+((c).*(Y.* (2.5 - 1.5.*Y + 0.25.*X)));
L = sqrt((F1/3).^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);
hold off

% The critical points are:
% (0,0) is a nodal source.
% (0, 8/5) is a saddle
% (1,0) is also a saddle
% (24/37, 52/37) is a nodal sink
% One of the species has evolved more than the other one causing the curve
% from c= 1/3 to widen more at the base. The more evolved species increases
% from (0,0) at a faster rate but their growth is limited because they
% eventually approach (24/37, 52/37) just like the other species.

c =

0.5000

Critical Points:
[ 0,   0]
[ 0,  8/5]
[ 1,   0]
[24/37, 52/37]
combining prob 5 and 6 with alpha =c=4/6

warning off all

c= 4/6

syms x y

sys1 = (((1-c)*(x*(1 - x - y)))+ ((c)*(x*(1 - x + 0.5*y))));
sys2 = ((((1-c)*(y*(1.5 - y - x)))+((c)*(y*(2.5 - 1.5*y + 0.25*x))));

[xc, yc] = solve(sys1, sys2, x, y);
disp ('Critical Points:'); disp ([xc yc])

f1 = @(t, x) [x(1)* (1 - x(1) - x(2)); x(2)* (1.5 - x(2) - x(1))];
f2 = @(t, x) [x(1)*(1 - x(1) + 0.5*x(2)); x(2)*(2.5 - 1.5*x(2) + 0.25*x(1))];

f3 = @(t, x) [(1-c)*(x(1)*(1 - x(1) - x(2)))+ (c*(x(1)*(1 - x(1) + 0.5*x(2))))];

figure; hold on
for a = 0.25: 0.75:3
    for b = 0.25: 0.25:3
        [t, xa] = ode45(f3, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))

        [t, xa] = ode45(f3, [0 -10], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end

title 'Problem 5 and 6 with c= 4/6'
axis ([0 2 0 2])

[X Y] = meshgrid (0:0.2:2, 0:0.25:2);

F1 = (((1-c).*X.* (1 - X - Y))+ (c).*X.* (1 - X + 0.5.*Y)));
F2 = (((1-c).*Y.* (1.5 - Y - X))+(c).*Y.* (2.5 - 1.5.*Y + 0.25.*X)));
L= sqrt((F1.^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);
hold off

% The critical points are:
% (0,0) is a nodal source.
% (0,13/8) is a saddle
% (1,0) is a also a saddle
% (1,1.5) is a nodal sink

% Both species have evolved a lot. They are getting close to a mutually
% benefiting relationship. They increase from (0,0) and approach (1,1.5).
% Both species also cannot keep increasing if they increase too much they
% decrease and approach (1, 1.5)

c =

0.6667

Critical Points:
[ 0, 0]
[ 0, 13/8]
[ 1, 0]
[ 1, 3/2]
combining prob 5 and 6 with alpha = $c = \frac{5}{6}$

warning off all

c = 5/6

syms x y

sys1 = (((1-c)*(x*(1 - x - y)))+ ((c)*(x*(1 - x + 0.5*y))));

sys2 = (((1-c)*(y*(1.5 - y - x)))+((c)*(y*(2.5 - 1.5*y + 0.25*x))));

[xc, yc] = solve(sys1, sys2, x, y);

disp ('Critical Points:'); disp ([xc yc])

f1 = @(t, x) [x(1)* (1 - x(1) - x(2)); x(2)* (1.5 - x(2) - x(1))];
\[ f_2 = @ (t, x) [x(1)*(1 - x(1) + 0.5*x(2)); x(2)*(2.5 - 1.5*x(2) + 0.25*x(1))]; \]
\[ f_3 = @ (t, x) [(((1-c)*(x(1)*(1 - x(1) - x(2))))+ (c*(x(1)*(1 - x(1) + 0.5*x(2))))); ((1-c)*(x(2)* (1.5 - x(2) - x(1))))+((c)*(x(2)*(2.5 - 1.5*x(2) + 0.25*x(1))))]; \]

figure; hold on

for a = 0.25: 0.75:3
    for b = 0.25: 0.25:3
        [t, xa] = ode45(f3, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f3, [0 -10], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end

title 'Problem 5 and 6 with c= 5/6'
axis ([0 2.5 0 2.5])

[X Y] = meshgrid (0:0.25:2.5, 0:0.5:2.5);
F1 = (((1-c).*(X.* (1 - X - Y)))+ ((c).*((X.* (1 - X + 0.5.*Y)))));
F2 = (((1-c).*(Y.* (1.5 - Y - X))+((c).*((Y.* (2.5 - 1.5.*Y + 0.25.*X)))));
L= sqrt((F1/3).^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);
hold off

% The critical points are:
% (0,0) is a nodal source.
% (0, 28/17) is a saddle
% (1,0) is a also a saddle
% (64/45, 76/45) is a nodal sink
% The relationship is evolving into a mutually benefiting one. They both
% increase from (0,0).
Both species have evolved to a point of almost
% tollerating one another without competing for resources. However, they
% always grow or decrease to approach (64/45, 76/45).

c =

0.8333

Critical Points:
[ 0, 0]
[ 0, 28/17]
[ 1, 0]
[ 64/45, 76/45]
combining prob 5 and 6 with alpha =c= 1

warning off all

c = 1

syms x y

sys1 = (((1-c)*(x*(1-x-y)))+ ((c)*(x*(1-x + 0.5*y))));

sys2 = (((1-c)*(y*(1.5 - y - x)))+((c)*(y*(2.5 - 1.5*y + 0.25*x))));

[xc, yc] = solve(sys1, sys2, x, y);
disp ('Critical Points:'); disp ([xc yc])

f1 = @(t,x) [x(1)* (1 - x(1) - x(2)); x(2)* (1.5 - x(2) - x(1))];
f2 = @ (t, x) [x(1)* (1 - x(1) + 0.5*x(2)); x(2)* (2.5 - 1.5*x(2) + 0.25*x(1))];
f3 = @ (t, x) [(((1-c)*(x(1)* (1 - x(1) - x(2))))+ (c*(x(1)* (1 - x(1) + 0.5*x(2))))); (((1-c)*(x(2)* (1.5 - x(2) - x(1))))+(c)*(x(2)* (2.5 - 1.5*x(2) + 0.25*x(1))))];
figure; hold on
for a = 0.25: 0.75:3
    for b = 0.25: 0.25:3
        [t, xa] = ode45(f3, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f3, [0 -10], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
end
title 'Problem 5 and 6 with c= 1'
axis ([0 4 0 4])
[X Y] = meshgrid (0:0.25:4, 0:0.5:4);
F1 = (((1-c).*(X.* (1 - X - Y)))+ ((c).*X.* (1 - X + 0.5.*Y)));
F2 = (((1-c).*(Y.* (1.5 - Y - X)))+((c).*Y.* (2.5 - 1.5.*Y + 0.25.*X)));
L= sqrt((F1/3).^2 + (F2/6).^2);
quiver(X, Y, F1./L, F2./L, 0.5);
hold off

% The critical points are:
% (0,0) is a nodal source.
% (0,5/3) is a saddle
(1,0) is a saddle
(2,2) is a nodal sink

The relationship between the species has evolved into a mutually beneficial one but there is still competition between individual species. An example of this is the trees in a forest and underground bacteria relationship. The trees compete for sunlight while the bacteria compete for food and shelter among themselves. They both benefit because the bacteria gets shelter and the trees can use waste produced by the bacteria. The graph is now an attracting cw spiral sink approaching (2,2) because there is still internal competition between the individual species.

c = 1

Critical Points:
[ 0, 0]
[ 0, 5/3]
[ 1, 0]
[ 2, 2]
Problem 5 and 6 with c = 1

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