MATLAB Assignment

Eq - $\frac{Dx^2}{Dt^2} + x^3 + cx = 0$
from $c=-1$ to $c=1$

```matlab
syms c
figure
c = -1
```
For c = -1, the curves from y=2 & y=-2 have an anti-node at x=-1 and then the next one at x=1.5, reaching max amplitude of 2 at x=0.4. The curves from y-1 & y=1 curve across horizontally almost linearly. The curve from y=0 opens up conically, is at its widest position at x=0.5 with an amplitude of 1.5, and then starts converging.
for x0 = -2:2
    for xp0 = -.5:0.1:.5
        [tfor, xfor] = ode45(rhs, [-2 2], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-2 2], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
hold off
title 'c=-.5';

Figure for c=-0.5, the curves from y=2 & y=-2 have the second anti-node a little earlier at x=1.1. The curves from y-1 & y=1 start converging and almost cross at x=2. The curve from y=0 opens up conically, and is at its widest a little earlier at x=0.4 with a decreased amplitude of 1.4, before starting to converge.

figure
c = -.1
rhs = @(t, x) [x(2); -x(1)^3 - c*x(1)];
[xa, ya] = ode45(rhs, [0 2], [-1 1]);
hold on
for x0 = -2:2
    for xp0 = -.5:0.1:.5

For c = -0.1, the curves from y=2 & y=-2 have the second anti-node much earlier at x=0.9. The curves from y=1 & y=-1 start converging much faster and cross at x=0. The curve from y=0 opens up conically, is at its widest even earlier at x=0.3 with a decreased amplitude of 1.

```matlab
figure
c = 0
rhs = @(t, x) [x(2); -x(1)^3 - c*x(1)];
[xa, ya] = ode45(rhs, [0 2], [-1 1]);
hold on
for x0 = -2:2
    for xp0 = -.5:.01:.5
        [tfor, xfor] = ode45(rhs, [-2 2], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-2 2], [x0 xp0]);
    end
end
```
For c= 0.0, the curves from y=2 & y=-2 have the second anti-node just a little before x=0.9. The curves from y-1 & y=1 start converging a little faster and cross right before x=0. The curve from y=0 opens up conically, is at its widest even earlier at x=0.3 with a decreased amplitude of 0.9.

```matlab
figure
c = .1
rhs = @(t, x) [x(2); -x(1)^3 - c*x(1)];
[xa, ya] = ode45(rhs, [0 2], [-1 1]);
hold on
for x0 = -2:2
    for xp0 = -.5:0.1:.5
        [tfor, xfor] = ode45(rhs, [-2 2], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-2 2], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
```
For $c=0.1$, the curves from $y=2$ & $y=-2$ have the second anti-node just a little before at $x=0.8$. The curves from $y=1$ & $y=-1$ start converging a little faster and cross right before $x=-0.2$. The curve from $y=0$ opens up conically, even more narrowly, being at its widest even earlier at $x=0.1$ with an amplitude of only $0.7$, and almost has an anti-node at $x=2$. 

```matlab
deficit = title('c=.1');
figure
c = .5
rhs = @(t, x) [x(2); -x(1)^3 - c*x(1)];
[xa, ya] = ode45(rhs, [0 2], [-1 1]);
hold on
for x0 = -2:2
    for xp0 = -5:0.1:.5
        [tfor, xfor] = ode45(rhs, [-2 2], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-2 2], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
```
For $c = 0.5$, the curves from $y=2$ & $y=-2$ have the second anti-node even earlier at $x=0.5$. The first anti-node also came earlier at $x=-1.2$ but the wavelength is still smaller than when $c < 0.5$. The curves from $y=1$ & $y=-1$ start converging way faster and cross at $x=-0.6$. The curve from $y=0$ opens up conically, very narrowly, being at its widest even earlier at $x=-0.2$, with an amplitude of 0.5, and has an anti-node at $x=2.0$.

```matlab
figure
c = 1
rhs = @(t, x) [x(2); -x(1)^3 - c*x(1)];
[xa, ya] = ode45(rhs, [0 2], [-1 1]);
hold on
for x0 = -2:2
    for xp0 = -.5:0.1:.5
        [tfor, xfor] = ode45(rhs, [-2 2], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-2 2], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
hold off
title 'c=1';
```
For \( c = 1.0 \), the curves from \( y=2 \) & \( y=-2 \) create the first and the second anti-node at the earliest yet at \( x=-1.3 \) and \( x=0.4 \) respectively. Both the curves have the smallest wavelength also. The curves from \( y=1 \) & \( y=1 \) start converge the fast yet also, crossing each other at \( x=-0.8 \). The curve from \( y=0 \) opens up conically, most narrowly, being at its widest even earlier at \( x=-0.5 \), with an amplitude of lower than 0.5. It has an anti-node even before \( x=1 \), therefore having the smallest wavelength yet.