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Extra credit project

```matlab
% Investigation of \( H(x,y) = \frac{1}{2}y^2 + 1 + \cos(x) \)
% \( \frac{d}{dt}(x; y) = (Hy; -Hx - (\delta)*Hy) \) as \( \delta \) moves from 1 to 0

\textbf{\( \delta = 1 \)}

\begin{verbatim}
f = @(t, x) [x(2); sin(x(1)) - x(2)];
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2));
    end
end
axis([-4.5 4.5 -4 4])
[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel('x')
ylabel('y')
title('Phase portrait and direction field when \( \delta = 1 \)')
\end{verbatim}
```
delta = 0.8

\[ f = \theta(t, x) [x(2); \sin(x(1)) - 0.8x(2)]; \]
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        \[ [t, xa] = \text{ode45}(f, [0 4], [a b]); \]
        plot(xa(:,1), xa(:,2))
        \[ [t, xa] = \text{ode45}(f, [0 -4], [a b]); \]
        plot(xa(:,1), xa(:,2))
    end
end
axis([-4.5 4.5 -4 4])

\[ [X,Y] = \text{meshgrid}(-4.5:.5:4.5, -4:.5:4); \]
\[ U = Y; \]
\[ V = \sin(X) - 0.8*Y; \]
\[ L = \sqrt{U.^2 + V.^2}; \]
\text{quiver}(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.8'
delta = 0.6

\[
f = @(t, x) \begin{bmatrix} x(2); \sin(x(1)) - 0.6 \times x(2) \end{bmatrix};
\]

figure, hold on
for a = -3:0.7:3
    for b = -3:0.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2));
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2));
    end
end
axis([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - 0.6*Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.6'
delta = 0.4

\[ f = \theta(t, x) \begin{bmatrix} x(2) \\ \sin(x(1)) - 0.4x(2) \end{bmatrix}; \]

figure, hold on
for a = -3:.7:3
  for b = -3:.7:3
    [t, xa] = ode45(f, [0 4], [a b]);
    plot(xa(:,1), xa(:,2))
    [t, xa] = ode45(f, [0 -4], [a b]);
    plot(xa(:,1), xa(:,2))
  end
end
axis([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - 0.4*Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.4'
delta = 0.2

\[
f = @(t, x) [x(2); \sin(x(1)) - .2*x(2)];
\]

figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - .2*Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.2'
delta = 0

\[ f(t, x) = [x(2); \sin(x(1))] \]

```matlab
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2));
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2));
    end
end
axis([-4.5 4.5 -4 4])
[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X);
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel('x')
ylabel('y')
title('Phase portrait and direction field when delta = 0')
```
Observations

% There is a very interesting progression in the phase portraits for the
% system as delta goes to 0. For delta = 1, the origin is an unstable
% saddle point and as the solution curves move away, they are attracted to
% the critical points (-3.14, 0) and (3.14, 0), as they are clockwise
% spiral sinks.

% When delta = 0.8, the saddle point, and spiral sinks remain the same, but
% the sinks are more attracting and there are more solution curves drawn
% in.

% When delta = 0.6, the saddle point begins to appear more horizontal, as
% does the whole phase portrait. Again, more solution curves are being
% directed about the clockwise spiral sink point.

% When delta = 0.4, the phase portrait is even more horizontal, and the
% slopes far away from the critical points are shifting away from the
% saddle point and more towards the sinks.

% When delta = 0.2, the phase portrait is almost completely horizontal, and
% the spiral sinks are becoming more like centers.

% When delta = 0, the origin is still a saddle point, and the points
% (-3.14, 0) and (3.14, 0) are stable clockwise centers. The system
% is closed and as a whole moves in the clockwise direction.