SERGE MUYA
Math246
Extra-credit

Phase Plane portrait VS contour of function

-MATRIX A
\[ a_{11} = \mu + 1 \ , \ a_{12} = 1 \ , \ a_{21} = 3 \ , \ a_{22} = \mu - 1 \ , \]
-Function: \[ h(x,y) = 3x^2 - 2xy - y^2 \]

1) \( \mu = -3 \)
\[ \text{ivp} = \begin{cases} Dx = -2x + 3y, & Dy = x - 4y, \\ x(0) = a, & y(0) = b \end{cases}; \]
\[ [x, y] = \text{dsolve}(	ext{ivp}, 't'); \]
\[ \text{xf} = @(t, a, b) \text{eval}(	ext{vectorize}(x)); \]
\[ \text{yf} = @(t, a, b) \text{eval}(	ext{vectorize}(y)); \]
\[ \text{figure; hold on} \]
\[ \text{t} = -3:0.1:3; \]
\[ \text{for } a = -2:2 \]
\[ \quad \text{for } b = -2:2 \]
\[ \quad \quad \text{plot}([\text{xf}(t, a, b), \text{yf}(t, a, b)]; \]
\[ \quad \end{end} \]
\[ \text{end} \]
\[ \text{axis([-5 5 -5 5])} \]
\[ [X, Y] = \text{meshgrid}(-5:0.1:5, -5:0.1:5); \]
\[ \text{hold on} \]
\[ \text{contour}(X, Y, 3.*X.^2 - 2.*X.*Y - Y.^2); \]
\[ \text{title 'Graph 1'} \]
\[ \text{hold off} \]

Here we have \( \mu = -3 \). By changing \( \mu \), the matrix \( A \) also changes thereby resulting in a unique phase portrait that is plotted in the same graph as the function \( h(x, y) \). This superimposition reveals that these two plots are not related. The graph of \( A \) is a nodal portrait.
2) \( \mu = -1 \)

\[ \text{ivp='Dx=3*y, Dy=x-2*y, x(0)=a, y(0)=b';} \]

\[ [x, y]=dsolve(ivp, 't'); \]

\[ xf= @(t, a, b) \text{eval(vectorize(x));} \]

\[ yf= @(t, a, b) \text{eval(vectorize(y));} \]

\[ \text{figure; hold on} \]

\[ t=-3:0.1:3; \]

\[ \text{for } a=-2:2 \]

\[ \quad \text{for } b=-2:2 \]

\[ \quad \text{plot(xf(t, a, b),yf(t, a, b));} \]

\[ \quad \text{end} \]

\[ \text{end} \]

\[ \text{axis([-5 5 -5 5])} \]

\[ [X,Y]=\text{meshgrid(-5:0.1:5, -5:0.1:5);} \]

\[ \text{hold on} \]

\[ \text{contour(X, Y, 3.*X.^2 - 2.*X.*Y- Y.^2);} \]

\[ \text{title 'Graph 2'} \]

\[ \text{hold off} \]

For this superimposition, \( \mu \) is -1; which resulted in a phase portrait of matrix \( A \) that is nodal. The phase portrait and the graph of the function are both nodal.
3) \( \mu = 0 \)

\[
\text{ivp} = \begin{align*}
&Dx = 1 \cdot x + 3 \cdot y, \\
&Dy = x - 1 \cdot y, \\
&x(0) = a, \\
&y(0) = b
\end{align*}
\]

\([x, y] = \text{dsolve(ivp, 't')}\);

\(xf = @(t, a, b) \text{eval(vectorize(x))};\)

\(yf = @(t, a, b) \text{eval(vectorize(y))};\)

figure; hold on

t = -3:0.1:3;

for \( a = -2:2 \)
  for \( b = -2:2 \)
    plot(xf(t, a, b), yf(t, a, b));
  end
end

hold off

axis([-5 5, -5 5])

\([X, Y] = \text{meshgrid}(-5:0.1:5, -5:0.1:5);\)

hold on

\( \text{contour(X, Y, 3 \cdot X^2 - 2 \cdot X \cdot Y - Y^2)};\)

\( \text{title('Graph 3')}\)

hold off

Here \( \mu \) is 0. With \( \mu = 0 \), the phase portrait of A and the graph of the function have closely looking graphs.
4) \( \mu = 2 \)

\[
\text{ivp} = \begin{align*}
\text{Dx} &= 1 \cdot x + 3 \cdot y, \\
\text{Dy} &= x - 1 \cdot y, \\
\text{x(0)} &= a, \\
\text{y(0)} &= b \\
\end{align*}
\]

\[
[x, y] = \text{dsolve}(\text{ivp}, 't');
\]

\[
\text{xf} = @(t, a, b) \text{eval(\text{vectorize(x))));}
\]

\[
\text{yf} = @(t, a, b) \text{eval(\text{vectorize(y));}
\]

figure; hold on

t = -3:0.1:3;

for \( a = -2:2 \)

for \( b = -2:2 \)

plot(xf(t, a, b), yf(t, a, b));

end

end

hold off

axis([-5 5 -5 5])

[X, Y] = meshgrid(-5:0.1:5, -5:0.1:5);

hold on

contour(X, Y, 3*X.^2 - 2*X.*Y - Y.^2);

title('Graph 4')

hold off

Here \( \mu = 2 \). The graphs of the matrix A and of the function \( h(x, y) \) are getting more closely aligned with each others.
5) $\mu = 3$

\[ \text{ivp='Dx=4*x+3*y, Dy=x-2*y, x(0)=a, y(0)=b';} \]

\[ [x, y]=\text{dsolve(ivp, 't');} \]

\[ \text{xf=} @(t, a, b) \text{eval(vectorize(x));} \]

\[ \text{yf=} @(t, a, b) \text{eval(vectorize(y));} \]

figure; hold on

\[ t=-3:0.1:3; \]

for $a=-2:2$

    for $b=-2:2$
        \[ \text{plot(xf(t, a, b),yf(t, a, b));} \]
    end
end

hold off

axis([-5 5 -5 5])

\[ [X,Y]=\text{meshgrid(-5:0.1:5, -5:0.1:5);} \]

hold on

\[ \text{contour(X, Y, 3.*X.^2 - 2.*X.*Y- Y.^2);} \]

\[ \text{title 'Graph 5';} \]

hold off

This is the last change of $\mu$ that also changes matrix $A$. As we can see in the phase portrait and the graph have closely similar graphs. Although, this is not perfect, the two graphs are shown to be related in some way.