Z = 0

```matlab
warning off all
syms x y q Z; Z = 0;
q = 5;
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -q:q
    for b = -q:q
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 0'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])
sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 0:'); disp([xc yc])
A = jacobian([sys1 sys2], [x y])
evals = eig(A);
disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (1/2,2):');
disp(double(subs(evals, {x y}, {(1/2), 2})))

% (0,0) is saddle point and is unstable.
% (1/2, 2) is a centers and is in the ccw direction and stable.

% As there becomes more predators(y becomes larger)
% the prey decreases(x becomes smaller). This is a continuous cycle. Once
% there becomes more prey, the predators become abundant and so therefore
% the prey diminishes.
```

Critical points for Z = 0:

```
[ 0, 0]
[ 1/2, 2]
```

```
A =
```
```
```
```
```
```
Eigenvalues at (0,0):
1.0000
-0.2500

Eigenvalues at (1/2,2):
0.0000 + 0.5000i
-0.0000 - 0.5000i

Z = 0.25

warning off all
syms x y Z; Z = 0.25;
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -5:5
  for b = -5:5
    [t, xa] = ode45(f, [0 5], [a b]);
    plot(xa(:,1), xa(:,2))
    [t, xa] = ode45(f, [0 -5], [a b]);
    plot(xa(:,1), xa(:,2))
  end
end
title 'Z = 0.25'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])
sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 0.25:'); disp([xc yc])
A = jacobian([sys1 sys2], [x y])
evals = eig(A);

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (8,0):');
disp(double(subs(evals, {x y}, {8, 0})))
disp('Eigenvalues at (1/2,15/8):');
disp(double(subs(evals, {x y}, {(1/2), (15/8)})))

% (0,0) continues to be an unstable saddle point.
% (8,0) is also an unstable saddle point.
% (1/2, 15/8) is still a centers and in the ccw direction and stable.

% The predator still depends on the prey, but has a slightly more difficult
time
% reproducing because of the saddle point at (8,0). This difficulty is
% only present for larger values of y.

Critical points for Z = 0.25:
[ 0, 0]
[ 8, 0]
[ 1/2, 15/8]

A =
[ 1-1/2*y-1/4*x, -1/2*x]
[ 1/2*y, -1/4+1/2*x]

Eigenvalues at (0,0):
1.0000
-0.2500

Eigenvalues at (8,0):
3.7500
-1.0000

Eigenvalues at (1/2,15/8):
-0.0312 + 0.4831i
-0.0313 - 0.4831i
warning off all
syms x y Z; Z = 0.5;
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -5:5
    for b = -5:5
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 0.5'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])

class = x*(1 - (1/2)*y - (Z/2)*x);
class2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(class, class2, x, y);
disp('Critical points for Z = 0.5:')
disp([xc yc])
A = jacobian([class class2], [x y]);
evals = eig(A);
disp('Eigenvalues at (0,0):')
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (4,0):')
disp(double(subs(evals, {x y}, {4, 0})))
disp('Eigenvalues at (1/2,7/4):')
disp(double(subs(evals, {x y}, {(1/2), (7/4)})))

% (0,0) is still an unstable saddle point.
% (4, 0) is also an unstable saddle point.
% (1/2, 7/4) is a ccw centers and is stable.
% In the predator prey model, the predator(y) depends on x. It is a
% continuous cycle and the prey does not have a difficult time reproducing
% for smaller values of y.

critical points for Z = 0.5:
[ 0, 0]
[ 4, 0]
[ 1/2, 7/4]

A =
[ 1-1/2*y-1/2*x, -1/2*x]
[ 1/2*y, -1/4+1/2*x]

Eigenvalues at (0,0):
1.0000
-0.2500

Eigenvalues at (4,0):
1.7500
-1.0000

Eigenvalues at (1/2,7/4):
-0.0625 + 0.4635i
-0.0625 - 0.4635i

Z = 0.75
warning off all
syms x y Z; Z = 0.75;
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -5:5
    for b = -5:5
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 0.75'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])
sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 0.75:'); disp([xc yc])
A = jacobian([sys1 sy2], [x y])
disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (8/3,0):');
disp(double(subs(evals, {x y}, {(8/3), 0})))
disp('Eigenvalues at (1/2,13/8):');
disp(double(subs(evals, {x y}, {(1/2), (13/8)}))))
% (0,0) is an unstable saddle point.
% (8/3,0) is an unstable saddle point.
% (1/2, 13/8) is a stable, ccw centers.

% Here the prey(x) has more difficulty in reproducing once the prey has
% diminished the prey's population. This is due to the fact that the
% centers point is nearing the unstable saddle at (0,0) and the saddle
% point at (8/3, 0)
Z = 1

warning off all
syms x y Z; Z = 1;
figure; hold on
f = @(t, x) [x(1) - (1/2)*x(2) - (Z/2)*x(1); x(2)*(-1/4) + (1/2)*x(1)];
for a = -5:5
    for b = -5:5
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 1'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])

sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 1:\n'); disp([xc yc])
A = jacobian([sys1 sys2], [x y])
evals = eig(A);
disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (2,0):');
disp(double(subs(evals, {x y}, {2, 0})))
disp('Eigenvalues at (1/2,3/2):');
disp(double(subs(evals, {x y}, {(1/2), (3/2)})))

% (0,0) is an unstable saddle.


\( (2, 0) \) is an unstable saddle.
\( (1/2, 3/2) \) is a stable ccw centers.

In this model the predator\( (y) \) still depends on \( x \). The prey\( (x) \) has a very
difficult time reproducing once it nears 0 because of how close the
centers is to the origin\( (\text{an unstable saddle}) \) and the saddle point at
\( (2, 0) \).

Critical points for \( Z = 1 \):
\[
\begin{array}{c}
0, 0 \\
2, 0 \\
1/2, 3/2 \\
\end{array}
\]

\[
A = 
\begin{bmatrix}
1 -1/2^*y - x, & -1/2^*x \\
1/2^*y, & -1/4 + 1/2^*x
\end{bmatrix}
\]

Eigenvalues at \( (0,0) \):
1.0000
-0.2500

Eigenvalues at \( (2,0) \):
0.7500
-1.0000

Eigenvalues at \( (1/2,3/2) \):
-0.1250 + 0.4146i
-0.1250 - 0.4146i

Analysis

\( (0,0) \) is a critical point for all graphs, and is a saddle point.
Therefore the origin is unstable for all the graphs.  

When Z = 0, the other critical point is a ccw centers. Once Z > 0, this critical point becomes a spiral sink and continues to be stable and ccw. Its x-coordinate is 1/2 for 0 => Z => 1 and the y-coordinate decreases as Z --> 1.

The third critical point appears once Z > 0. It is a saddle point also, like (0,0) and has a constant y-coordinate of 0, while its x-coordinate decreases as Z --> 1.

For 0 => Z => 1, a_21 > 0 so all critical points are in the ccw direction.

y is the predator and x is the prey in this predator-prey system. For Z = 0, as there becomes more predators (y becomes larger) and so therefore the prey decreases (x becomes smaller). This is a continuous cycle. Once there becomes more prey, the predators become abundant and so therefore the prey diminishes.

This pattern continues as Z increases towards 1. When Z = 1, y (the predator) still depends on x (the prey), but now the prey (x) decreases as y increases along with when x increases. The addition of the saddle point along the x-axis causes this.

The predator (y) becomes larger when x becomes smaller because the predators are eating all of their food (x).

The predator-prey model is a continuous loop of events (prey becomes abundant, predators eat most of prey, predators decrease in population until the prey becomes abundant again), while Z is closer to 0. As Z = 1, the prey do not reproduce for large values of y (predators).