(1) [5] The 2×2 matrix $A$ has the real eigenpairs $\begin{pmatrix} 1, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{pmatrix}$ and $\begin{pmatrix} 3, \begin{pmatrix} -1 \\ 2 \end{pmatrix} \end{pmatrix}$.

(a) Sketch a phase portrait for the system $\frac{dx}{dt} = Ax$. Indicate typical trajectories.
(b) For each portrait identify its type and state whether the origin is attracting, stable, unstable, or repelling.

**Solution:** Because the eigenvalues of $A$ are real and of have the same sign, the phase portrait is a *nodal source*. Because all trajectories move away from the origin as $t$ increases, it is *repelling* (and therefore also *unstable*).

- Your phase portrait should indicate trajectories on the line $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ move away from the origin with arrows.
- It should indicate trajectories on the line $c_2 \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ move away from the origin with double arrows.
- It should also indicate a family of trajectories move away from the origin along curves that emerge from the origin tangent to the line $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and further away become more parallel to the line $c_2 \begin{pmatrix} -1 \\ 2 \end{pmatrix}$.

(2) [5] Let $A = \begin{pmatrix} -2 & 5 \\ -2 & 0 \end{pmatrix} x$.

(a) Sketch a phase portrait for the system $\frac{dx}{dt} = Ax$. Indicate typical trajectories.
(b) For each portrait identify its type and state whether the origin is attracting, stable, unstable, or repelling.

**Solution:** The characteristic polynomial of $A$ is

$$p(z) = z^2 - \text{tr}(A)z + \det(A) = z^2 + 2z + 10 = (z + 1)^2 + 3^2.$$  

The eigenvalues of $A$ are therefore $-1 + i3$ and $-1 - i3$, whereby the phase portrait is a *spiral sink*. Because $a_{21} = -2 < 0$ the spiral will be *clockwise*. Because all trajectories move towards the origin as $t$ increases, it is *attracting* (and therefore also *stable*).

Your phase portrait should indicate a family of trajectories that spiral into the origin in a clockwise fashion.