Quiz 6 Solutions, Math 220, Professor David Levermore
Friday, 8 October 2010

(1) [5] A rectangular garden of area 80 square feet is to be surrounded on three sides by a brick wall costing 10 dollars per foot and on one side by a fence costing 6 dollars per foot. Find the dimensions of the garden that minimizes the cost of the materials.

Solution. Let the dimensions of the rectangle by x by y where x is the length (in feet) of the fence and one brick wall, while y is the length of two brick walls. The objective is the cost of materials, which is

$$C = 16x + 20y\,,$$

while the constraint is $xy = 80$. Solving the constraint for y gives $y = 80/x$, so that the cost can be expressed as

$$C = 16x + 20\frac{80}{x}\,.$$

The critical points are found by solving

$$0 = \frac{\mathrm{d}C}{\mathrm{d}x} = 16 - \frac{1600}{x^2}\,,$$

whereby $x^2 = 100$. Hence, $x = 10$ and $y = 80/10 = 8$.

(2) [5] A distributor of sporting goods expects to sell 12,000 cases of tennis balls at a steady rate over the coming year. Yearly carrying costs are 15 dollars per case, while the cost of placing an order with the manufacturer is 100 dollars. How many orders should be placed to minimize the inventory cost?

Solution. If n orders are placed the carrying cost of the inventory will be $15 \cdot \frac{1}{2}12,000/n$, while net cost of placing the orders will be $100n$. The total inventory cost as a function of n will therefore be

$$C(n) = 15\frac{6000}{n} + 100n\,.$$

The critical points are found by setting

$$0 = C'(n) = -15\frac{6000}{n^2} + 100\,,$$

whereby $n^2 = 15 \cdot \cdot \cdot 60 = 900$. Hence $n = 30$. This is clearly a minimum because $C'(n)$ changes sign from negative to positive. Therefore 30 orders should be placed during the year in order to minimize the inventory cost.