Quiz 8 Solutions, Math 220, Professor David Levermore
Friday, 22 October 2010

(1) [3] Find the first derivative of $h(t) = \dfrac{e^{t^3} + e^{-t^3}}{e^{t^2}}$.

Solution. The good way to do this is to first simplify $h(t)$ as

$$h(t) = \frac{e^{t^3} + e^{-t^3}}{e^{t^2}} = \big(e^{t^3} + e^{-t^3}\big)e^{-t^2} = e^{t^3-t^2} + e^{-t^3-t^2} \,.$$

Then the exponential rule gives

$$h'(t) = e^{t^3-t^2}(3t^2 - 2t) + e^{-t^3-t^2}(-3t^2 - 2t) \,.$$

Alternative Solution. Another way to do this is to simplify $h(t)$ as

$$h(t) = \frac{e^{t^3} + e^{-t^3}}{e^{t^2}} = \big(e^{t^3} + e^{-t^3}\big)e^{-t^2} \,.$$

The the product and exponential rules give

$$h'(t) = \big(e^{t^3}3t^2 + e^{-t^3}(-3t^2)\big)e^{-t^2} + \big(e^{t^3} + e^{-t^3}\big)\big(e^{-t^2}(-2t)\big) \,.$$

Another Alternative Solution. The quotient and exponential rules give

$$h'(t) = \frac{\big(e^{t^3}3t^2 + e^{-t^3}(-3t^2)\big)e^{t^2} - \big(e^{t^3} + e^{-t^3}\big)\big(e^{t^2}2t\big)}{e^{2t^2}} \,.$$

(2) [3] Solve $\big(2^{x+1} \cdot 2^{-3}\big)^2 = 4$ for x.

Solution. Because

$$\big(2^{x+1} \cdot 2^{-3}\big)^2 = \big(2^{x+1-3}\big)^2 = \big(2^{x-2}\big)^2 = 2^{2(x-2)} = \big(2^2\big)^{x-2} = 4^{x-2} \,,$$

you see that $4^{x-2} = 4$. Therefore $x - 2 = 1$, whereby $x = 3$.

Alternative Solution. Because $4 = 2^2$, the equation is equivalent to $2^{x+1} \cdot 2^{-3} = 2$. But $2^{x+1} \cdot 2^{-3} = 2^{x+1-3} = 2^{x-2}$, so that $x - 2 = 1$, whereby $x = 3$.

(3) [4] Find the point x where the graph of $y = (1 + x^2)e^x$ has a horizontal tangent line.

Solution. The tangent line will be horizontal where the derivative is zero. By the product and exponential rules

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2xe^x + (1 + x^2)e^x = (x^2 + 2x + 1)e^x = (x + 1)^2 e^x \,.$$

Because $e^x > 0$ for every x, this derivative is zero only when $(x + 1)^2 = 0$, which happens at the point $x = -1$.