Quiz 3 Solutions, Math 246, Professor David Levermore
Tuesday, 15 February 2011

(1) [4] Consider the following MATLAB function M-file.

```
function [t,y] = solveit(ti, yi, tf, n)

h = (tf - ti)/n;
t = zeros(n + 1, 1);
y = zeros(n + 1, 1);
t(1) = ti;
y(1) = yi;
for j = 1:n
t(j + 1) = t(j) + h;
y(j + 1) = y(j) + h*(t(j) - y(j)^3);
end
```

(a) What initial-value problem is being approximated numerically?
(b) Identify the numerical method being used and give its order.

Solution (a). The initial-value problem being approximated is

$$\frac{\mathrm{d}y}{\mathrm{d}t} = t - y^3 , \qquad y(\mathrm{ti}) = \mathrm{yi} .$$

Solution (b). The *explicit (or forward) Euler* method is used, which is *first-order.*

(2) [6] A student borrows \$6000 at an interest rate of 5% per year that is compounded continuously. Assume that the student makes payments continuously at a constant rate of \$1200 per year. Let $B(t)$ denote the balance of the loan at t years.
(a) Write down an initial-value problem that governs $B(t)$ for so long as $B(t) \geq 0$.
(b) Solve the initial-value problem for $B(t)$.

Solution (a). The initial-value problem that governs $B(t)$ is

$$\frac{\mathrm{d}B}{\mathrm{d}t} = .05B - 1200 , \qquad B(0) = 6000 .$$

Solution (b). The differential equation is *linear.* Its normal form is

$$\frac{\mathrm{d}B}{\mathrm{d}t} - .05B = -1200 .$$

Its integrating factor form is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(e^{-.05t}B\right) = -1200e^{-.05t} .$$

Integrating both sides yields

$$e^{-.05t}B = \tfrac{-1200}{-.05}e^{-.05t} + c = 24,000e^{-.05t} + c .$$

Imposing the initial condition gives $6000 = B(0) = 24,000 + c$, whereby $c = -18,000$. The solution is therefore

$$B(t) = 24,000 - 18,000e^{.05t} .$$