Quiz 2 Solutions, Math 246, Professor David Levermore
Tuesday, 7 February 2012

(1) [5] Find the solution of the initial-value problem

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{3t^2}{2w}\,, \qquad w(0) = w_I \quad \text{for some } w_I < 0\,.$$

Give its interval of definition in terms of w_I.

Solution. This equation is separable. Its separated differential form is $2w\,\mathrm{d}w = 3t^2\,\mathrm{d}t$, whereby

$$\int 2w\,\mathrm{d}w = \int 3t^2\,dt\,.$$

By integrating both sides we find that $w^2 = t^3 + c$. The initial condition then implies that $w_I^2 = 0^3 + c$, whereby $c = w_I^2$. The solution is thereby given implicity by

$$w^2 = t^3 + w_I^2\,.$$

Whenever $t^3 + w_I^2 > 0$ this can be solved explicitly for w to obtain

$$w = -\sqrt{t^3 + w_I^2}\,,$$

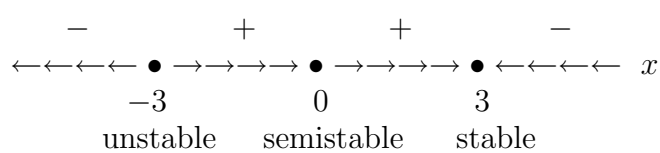
where the negative square root is taken because $w_I < 0$. The condition $t^3 + w_I^2 > 0$ implies that $t^3 > -w_I^2$, which implies that $t > -w_I^{\frac{2}{3}}$. Therefore the interval of definition is $(-w_I^{\frac{2}{3}}, \infty)$.

(2) [5] Sketch the phase-line portrait for the equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -x^2(x-3)(x+3)e^x\,.$$

Identify every stationary (equilibrium) point as either stable, unstable, or semistable. (You do not have to find the solution!)

Solution. The stationary points are found by setting

$$0 = -x^2(x-3)(x+3)e^x\,.$$

The stationary points are therefore $x = 0$, $x = 3$, and $x = -3$. The phase-line portrait is