Quiz 3 Solutions, Math 246, Professor David Levermore
Tuesday, 14 February 2012

(1) [4] Consider the following MATLAB function M-file.

```
function [t,y] = solveit(ti, yi, tf, n)

t = zeros(n + 1, 1); y = zeros(n + 1, 1);
t(1) = ti; y(1) = yi; h = (tf - ti)/n;
for j = 1:n
s = t(j) + h/2; x = y(j) + h*(t(j)^2 + y(j)^3)/2;
t(j + 1) = t(j) + h; y(j + 1) = y(j) + h*(s^2 + x^3);
end
```

(a) What initial-value problem is being approximated numerically?
(b) Identify the numerical method being used and give its order.

Solution: The initial-value problem being approximated is

$$\frac{dy}{dt} = t^2 + y^3 \,, \qquad y(\mathrm{ti}) = \mathrm{yi} \,.$$

The Heun-Midpoint (modified Euler) method is being used, which is second order.

(2) [2] Suppose you are using the Runge-Kutta method to numerically approximate the solution of an initial-value problem over the time interval $[0, 3]$. By what factor would you expect the global error to decrease when the number of time steps that you take is increased from 600 to 1800.

Solution: When you increase the number of time steps by a factor of 3, the time step h is reduced by a factor of 1/3. Because the Runge-Kutta method is fourth order, the error is therefore reduce by a factor of $1/3^4 = 1/81$.

(3) [4] Consider the initial-value problem

$$\frac{dv}{dt} = v^2 - 3v \,, \qquad v(0) = 4 \,.$$

Use the Heun-midpoint method with $h = .1$ to approximate $v(.1)$. Leave your answer as an arithmetic expression.

Solution: Set $v_0 = v(0) = 4$. The Heun-midpoint method then yields

$$\begin{aligned} v_{\frac{1}{2}} &= v_0 + h\big(v_0^2 - 3v_0\big)/2 = 4 + .1\big(4^2 - 3 \cdot 4\big)/2 \\ &= 4 + .1\big(16 - 12\big)/2 = 4.2 \,, \\ v(.1) \approx v_1 &= v_0 + h\big(v_{\frac{1}{2}}^2 - 3v_{\frac{1}{2}}\big) = 4 + .1\big((4.2)^2 - 3 \cdot 4.2\big) \,. \end{aligned}$$