

Sample Problems for Second In-Class Exam
Math 246, Fall 2012, Professor David Levermore

- (1) Give the interval of definition for the solution of the initial-value problem

$$\frac{d^3x}{dt^3} + \frac{\cos(3t)}{4-t} \frac{dx}{dt} = \frac{e^{-2t}}{1+t}, \quad x(2) = x'(2) = x''(2) = 0.$$

- (2) Suppose that $Y_1(t)$ and $Y_2(t)$ are solutions of the differential equation

$$y'' + 2y' + (1 + t^2)y = 0.$$

Suppose you know that $W[Y_1, Y_2](0) = 5$. What is $W[Y_1, Y_2](t)$?

- (3) The function $Y(t) = t$ is a solution of the differential equation

$$(t^2 + 4)y'' - 2ty' + 2y = 0.$$

Find a general solution of this equation.

- (4) Show that the functions $Y_1(t) = \cos(t)$, $Y_2(t) = \sin(t)$, and $Y_3(t) = 1$ are linearly independent.

- (5) Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (listed with their multiplicities) are $-2 + i3$, $-2 - i3$, $i7$, $i7$, $-i7$, $-i7$, 5 , 5 , 5 , -3 , 0 , 0 .

(a) Give the order of L .

(b) Give a general real solution of the homogeneous equation $Ly = 0$.

- (6) Give the natural fundamental set of solutions associated with $t = 0$ for each of the following equations.

(a) $y'' - 6y' + 9y = 0$.

(b) $y'' + 4y' + 20y = 0$.

- (7) Let $D = \frac{d}{dt}$. Solve each of the following initial-value problems.

(a) $D^2y + 4Dy + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

(b) $D^2y + 9y = 20e^t$, $y(0) = 0$, $y'(0) = 0$.

- (8) Let $D = \frac{d}{dt}$. Give a general real solution for each of the following equations.

(a) $D^2y + 4Dy + 5y = 3 \cos(2t)$.

(b) $D^2y - y = t e^t$.

(c) $D^2y - y = \frac{1}{1 + e^t}$.

- (9) Let $D = \frac{d}{dt}$. Consider the equation

$$Ly = D^2y - 6Dy + 25y = e^{t^2}.$$

- (a) Compute the Green function $g(t)$ associated with L .
 (b) Use the Green function to express a particular solution $Y_P(t)$ in terms of definite integrals.

- (10) The functions t and t^2 are solutions of the homogeneous equation

$$t^2 \frac{d^2y}{dt^2} - 2t \frac{dy}{dt} + 2y = 0 \quad \text{over } t > 0.$$

(You do not have to check that this is true!)

- (a) Compute their Wronskian.
 (b) Solve the initial-value problem

$$t^2 \frac{d^2y}{dt^2} - 2t \frac{dy}{dt} + 2y = t^3 e^t, \quad y(1) = y'(1) = 0, \quad \text{over } t > 0.$$

Try to evaluate all definite integrals explicitly.

- (11) What answer will be produced by the following MATLAB commands?

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>> ode1 = 'D2y + 2*Dy + 5*y = 16*exp(t)';
>> dsolve(ode1, 't')
ans =
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- (12) The vertical displacement of a mass on a spring is given by

$$h(t) = 4e^{-t} \cos(7t) - 3e^{-t} \sin(7t),$$

where positive displacements are upward.

- (a) Express $h(t)$ in the form $h(t) = Ae^{-t} \cos(\omega t - \delta)$ with $A > 0$ and $0 \leq \delta < 2\pi$, identifying the quasiperiod and phase of the oscillation. (The phase may be expressed in terms of an inverse trig function.)
 (b) Sketch the solution over $0 \leq t \leq 2$.

- (13) When a mass of 4 grams is hung vertically from a spring, at rest it stretches the spring 9.8 cm. (Gravitational acceleration is $g = 980 \text{ cm/sec}^2$.) At $t = 0$ the mass is displaced 3 cm above its equilibrium position and is released with no initial velocity. It moves in a medium that imparts a drag force of 2 dynes ($1 \text{ dyne} = 1 \text{ gram cm/sec}^2$) when the speed of the mass is 4 cm/sec. There are no other forces. (Assume that the spring force is proportional to displacement and that the drag force is proportional to velocity.)

- (a) Formulate an initial-value problem that governs the motion of the mass for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)
 (b) What is the natural frequency of the spring?
 (c) Show that the system is under damped and find its quasifrequency.