**Quiz 10 Solutions, Math 246, Professor David Levermore**
**Tuesday, 1 December 2015**

(1) [5] A $2 \times 2$ matrix $\mathbf{A}$ has the eigenpairs

$$\left(1\,, \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right)\,, \qquad \left(2\,, \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right)\,.$$

Sketch a phase-plane portrait that indicates typical orbits for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Identify its type. Classify the origin as either attracting, stable but not attracting, unstable but not repelling, or repelling.

**Solution.** Because $\mathbf{A}$ has two positive eigenvalues, the phase portrait is a *nodal source*. The origin is thereby *repelling*. The phase portrait should show one orbit that moves away from the origin along each half of the lines $y = \frac{1}{2}x$ and $y = -x$. The phase portrait should indicate that every other orbit emerges from the origin tangent to the line $y = \frac{1}{2}x$.

(2) [5] Sketch a phase-plane portrait that indicates typical orbits for the system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ -1 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\,.$$

Identify its type. Classify the origin as either attracting, stable but not attracting, unstable but not repelling, or repelling.

**Solution.** The characteristic polynomial of the given matrix $\mathbf{A}$ is

$$p(z) = z^2 - \mathrm{tr}(\mathbf{A})z + \det(\mathbf{A}) = z^2 + 6z + 13 = (z+3)^2 + 4\,.$$

We see that the mean $\mu = -3$ and the discriminant $\delta = -4$. Because $\delta = -4 < 0$, there are no real eigenpairs. Because $\mu = -3 < 0$, $\delta = -4 < 0$, and $a_{21} = -1 < 0$ the phase portrait is a *clockwise spiral sink*. The origin is thereby *attracting*. The phase portrait should indicate a family of clockwise spiral orbits that approach the origin.