Quiz 1 Solutions, Math 246, Professor David Levermore
Thursday, 2 February 2017

(1) [4] For each of the following ordinary differential equations, determine its order and whether it is linear or nonlinear. If it is nonlinear, circle a term that makes it so.

(a) $w''' + e^w w'' = 3w' + \sin(t)$ **Solution.** third order, nonlinear, $e^w w''$.

(b) $h^{(4)} + \cos(x)h'' = 3h + \cos(h)$ **Solution.** fourth order, nonlinear, $\cos(h)$.

(2) [4] Solve the initial-value problem

$$x\frac{\mathrm{d}v}{\mathrm{d}x} + 3v = 15x^2\,, \qquad v(-1) = 4\,.$$

Solution. This equation is linear. Its normal form is

$$\frac{\mathrm{d}v}{\mathrm{d}x} + \frac{3}{x}\,v = 15x\,.$$

An integrating factor is $e^{A(x)}$ where $A'(x) = 3/x$. Setting $A(x) = 3\log(x)$, we find that $e^{A(x)} = e^{3\log(x)} = x^3$. Hence, the problem has the intgrating factor form

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(x^3 v\big) = x^3 \cdot (15x) = 15x^4\,.$$

Integrating both sides yields

$$x^3 v = 3x^5 + c\,.$$

Imposing the initial condition gives

$$(-1)^3 \cdot 4 = 3 \cdot (-1)^5 + c\,,$$

whereby $c = -1$. Therefore the solution is

$$v = \frac{3x^5 - 1}{x^3}\,.$$

Remark. The interval of definition for this solution is $(-\infty, 0)$. Can you see why?

(3) [2] What is the interval of definition for the solution of the initial-value problem

$$\frac{\mathrm{d}u}{\mathrm{d}t} + \frac{\cos(t)}{t^2 - 16}\,u = \frac{\sin(t)}{t^2 - 4}\,, \qquad u(-3) = 2\,.$$

(You do not have to solve this equation to answer this question!)

Solution. This equation is linear and is already in normal form. The coefficient $\cos(t)/(t^2 - 16)$ is continuous everywhere except at $t = \pm 4$, where it is undefined. The forcing $\sin(t)/(t^2 - 4)$ is continuous everywhere except at $t = \pm 2$, where it is undefined. The initial time is $t = -3$. Therefore we can read off that the interval of definition for the solution is $(-4, -2)$ because:

- the initial time $t = -3$ is in $(-4, -2)$,
- the coefficient and forcing are both continuous over $(-4, -2)$,
- the coefficient is not defined at $t = -4$,
- the forcing is not defined at $t = -2$.