First In-Class Exam
Math 246, Professor David Levermore
Tuesday, 19 September 2017

Your Name: ______________________________

UMD SID: ______________________________

Discussion Instructor (circle one): Yan Tay Jing Zhou
Discussion Time (circle one): 8:00 9:00 10:00

No books, notes, calculators, or any electronic devices. If you need more space to answer a problem then use the back of one of these pages. Clearly indicate where your answer to each part of every problem is located. **Your reasoning must be given for full credit.** Any work that you do not want to be considered should be crossed out. Good luck!

University Honor Pledge: *I pledge on my honor that I have not given or received any unauthorized assistance on this examination.*

Signature: ______________________________

Problem 1:	________/8	Problem 2:	________/20
Problem 3:	________/12	Problem 4:	________/12
Problem 5:	________/6	Problem 6:	________/6
Problem 7:	________/8	Problem 8:	________/8
Problem 9:	________/20	Total Score:	________/100
		Grade:	________

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(1) [8] In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its current population such that it would triple every seven weeks. There are 210,000 mosquitoes in the area when a flock of birds arrives that eats 30,000 mosquitoes per week.

(a) [6] Write down an initial-value problem that governs $M(t)$, the population of mosquitoes in the area after the flock of birds arrives. (Do not solve the initial-value problem!)

(b) [2] Is the flock of birds large enough to control the mosquitoes?

(2) [20] Find an explicit solution for each of the following initial-value problems.

(a) $\dfrac{\mathrm{d}u}{\mathrm{d}t} = (2u - u^2)\,t\,, \qquad u(0) = 4\,.$

(b) $t\,\dfrac{\mathrm{d}y}{\mathrm{d}t} + 2y = \dfrac{2}{1+t^2}\,, \qquad y(1) = 0\,.$

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(3) [12] Consider the differential equation $\dfrac{\mathrm{d}w}{\mathrm{d}t} = \dfrac{(w+2)(2-w)(w-6)^2}{w+6}\, e^{-2w}$.

(a) [7] Sketch its phase-line portrait over the interval $-9 \le w \le 9$. Identify points where it has no solution. Identify its stationary points and classify each as being either stable, unstable, or semistable.

(b) [5] For each stationary point identify the set of initial values $w(0)$ such that the solution $w(t)$ converges to that stationary point as $t \to \infty$.

(4) [12] Consider the following MATLAB function M-file.

```
function [t,x] = solveit(tI, xI, tF, n)

t = zeros(n + 1, 1); x = zeros(n + 1, 1);
t(1) = tI; x(1) = xI; h = (tF − tI)/n; hhalf = h/2;
for i = 1:n
fnow = (t(i))ˆ2 + cos(t(i) * x(i));
tfull = t(i) + h; xfull = x(i) + h * fnow;
ffull = (tfull)ˆ2 + cos(tfull * xfull);
t(i + 1) = tfull; x(i + 1) = x(i) + hhalf * (fnow + ffull);
end
```

Suppose that the input values are tI = 1, xI = 0, tF = 10, and n = 90.

(a) [4] What initial-value problem is being approximated numerically?

(b) [2] What is the numerical method being used?

(c) [2] What is the step size?

(d) [4] What will be the output values of t(2) and x(2)?

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(5) [6] Give the interval of definition for the solution of the initial-value problem

$$\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{1}{\sin(t)}\, q = \frac{1}{t^2 - 16}\,, \qquad q(-5) = 3\,.$$

(You do not have to solve this equation to answer this question!)

(6) [6] Consider the initial-value problem

$$\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{5x^4}{4v^3}\,, \qquad v(-1) = v_I \quad \text{for some } v_I \text{ in } (-\infty, 0)\,.$$

The solution satisfies

$$v^4 = v_I^4 + 1 + x^5\,.$$

Gives its interval of definition as a function of v_I.

(7) [8] Suppose that a numerical method is used to approximate the solution of an initial-value problem over the time interval $[1, 5]$ with 1000 uniform time steps. About how many uniform time steps are needed to reduce the global error of the approximation by a factor of $\frac{1}{256}$ if the method used was each of the following? (Note $256 = 16^2 = 4^4$.)

(a) Runge-Kutta method
(b) Runge-trapezoidal method
(c) Runge-midpoint method
(d) Euler method

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(8) [8] A 2 gram (g) mass initially at rest is dropped in a medium that offers a resistance of $v^2/10$ dynes (= g cm/sec^2) where v is the downward velocity (cm/sec) of the mass. The gravitational acceleration is 980 cm/sec^2.

(a) [6] Write down an initial-value problem that governs v as a function of time. (Do not solve the initial-value problem!)

(b) [2] What is the terminal velocity of the mass?

(9) [20] For each of the following differential forms determine if it is exact or not. If it is exact then give an implicit general solution. Otherwise find an integrating factor. (Do not find a general solution in the last case.)

(a) $\big(\cos(x+y) + e^x\big)\,\mathrm{d}x + \big(\cos(x+y) - 3y^2\big)\,\mathrm{d}y = 0\,.$

(b) $(2xy + 4x^3)\,\mathrm{d}x + (x^2y + x^2 + x^4)\,\mathrm{d}y = 0\,.$