Fourth Homework: MATH 410
Due Tuesday, 26 September 2017

1. Prove Proposition 3.12 in the notes.
2. Let $\{a_k\}_{k\in\mathbb{N}}$ be a real sequence and $\{a_{n_k}\}$ be any subsequence. Show that
$$\sum_{k=0}^{\infty} a_k \quad \text{converges absolutely} \qquad \Longrightarrow \qquad \sum_{k=0}^{\infty} a_{n_k} \quad \text{converges absolutely}\,.$$
3. Give examples of both a divergent series and a convergent series such that
$$\limsup_{k\to\infty} \sqrt[k]{|a_k|} = 1\,.$$
4. Consider the set
$$\left\{ x \in \mathbb{R} \,:\, \sum_{n=0}^{\infty} \frac{(4n)!}{(2n)!}\,\frac{n!}{(3n)!}\,x^n \quad \text{converges} \right\}.$$
Use the root test to prove that this set is an interval and find its endpoints. You may use the fact that
$$\lim_{k\to\infty} \frac{\sqrt[k]{k!}}{k} = \frac{1}{e}\,.$$
5. Prove the divergence assertion of Proposition 3.15 in the notes. Show that if neither condition of Proposition 3.15 is satisfied then the series may either converge or diverge.
6. Prove Proposition 3.17 in the notes.
7. Consider the set
$$\left\{ x \in \mathbb{R} \,:\, \sum_{n=0}^{\infty} \frac{(4n)!}{(2n)!}\,\frac{n!}{(3n)!}\,x^n \quad \text{converges} \right\}.$$
Use the ratio test to prove that this set is an interval and find its endpoints.
8. Determine all $x, p \in \mathbb{R}$ for which the Fourier p-series
$$\sum_{k=1}^{\infty} \frac{\sin(kx)}{k^p} \quad \text{converges}\,.$$
9. Exercise 2 of Section 2.2 in the text.
10. Exercise 4 of Section 2.2 in the text.
11. Prove Proposition 4.1 in the notes.
12. Prove Proposition 4.2 in the notes.
13. Prove Proposition 4.6 in the notes.
14. Prove Proposition 4.8 in the notes.