What is a Theory of Human Linguistic Understanding?

#{F & G} > #{F} – #{F & G}

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Human Language:
a language of the sort that human children can naturally acquire

Davidson’s Conjecture:
for each Human Language L, there is a theory of truth that is also the core of an adequate theory of meaning for L

Dummett’s Question:
what is a theory of meaning for L a theory of?

Dummett’s Answer:
a theory of meaning for L is a theory of understanding for L, even if this is not a theory of truth-in-L

Next Question:
what is a theory of understanding for L?
**Old Idea:** (1) understanding expressions is a matter of *analyzing* them in terms of “basic” concepts;
(2) analytically basic concepts are *epistemically* special;
so (3) understanding is importantly related to *verification*

- this does *not* imply an implausible form of *verificationism*
- the Logical Form of a sentence is (among other things) a Verification Procedure that the sentence reflects; but this procedure need *not* be *epistemically special* or *always used*

  Bert arrived, and Ernie left  \[ P \land Q \]

  There is a red ball on the table  \[ \exists x (R_x \land B_x \land O_x) \]

- we don’t have *a priori* knowledge of when speakers will use a Logical Form as a Verification Procedure, as opposed to relying on ancillary beliefs and using an alternative procedure
Old Idea: (1) understanding expressions is a matter of analyzing them in terms of “basic” concepts; so (3) understanding is importantly related to verification

• we don’t have a priori knowledge of when speakers will use a Logical Form as a Verification Procedure, as opposed to relying on ancillary beliefs and using an alternative procedure

• but reliable verification under time pressure can be illuminating, given good theories of which procedures can be used in that window

• unreliable verification with no time pressure can also be illuminating: it tells against a candidate Logical Form for a sentence S if speakers have trouble evaluating sentence S in situations that make it easy to evaluate the candidate Logical Form of S.
Most of the dots are yellow

15 dots:
9 yellow
6 blue
‘Most of the dots are yellow’

\[ \#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT}\}/2 \quad 9 > 15/2 \]

More than half of the dots are yellow

\[ \#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT} \& \sim\text{YELLOW}\} \quad 9 > 6 \]

The yellow dots outnumber the nonyellow dots

There are more yellow dots than nonyellow dots

\[ \exists s : s \subset \{\text{DOT} \& \text{YELLOW}\}[\text{OneToOne}[s, \{\text{DOT} \& \sim\text{YELLOW}\}] \]

\[ \#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT}\} - \#\{\text{DOT} \& \text{YELLOW}\} \quad 9 > (15 - 9) \]

The number of yellow dots exceeds

the number of dots minus the number of yellow dots

true at the same possible worlds...provably so, given arithmetic...

but some analyses may be more “natural” than others
Before E: Church (1941, pp. 1-3) on Lambdas

**function in intension**

**computational procedure**

**function in extension**

**set of input-output pairs**

\[ |x - 1| + \sqrt{x^2 - 2x + 1} \]

\{(-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), \ldots \}

\[ \lambda x . |x - 1| = \lambda x . + \sqrt{x^2 - 2x + 1} \]

\[ \lambda x . |x - 1| \neq \lambda x . + \sqrt{x^2 - 2x + 1} \]

\[ \text{Extension}[\lambda x . |x - 1|] = \text{Extension}[\lambda x . + \sqrt{x^2 - 2x + 1}] \]

\[ \text{Extension}[\text{Glonker}(x)] = \text{Extension}[\text{Flonker}(x)] \]
Before E: Church (1941, pp. 1-3) on Lambdas

- Function in intension
- Computational procedure
- Function in extension
- Set of input-output pairs

Extension[$\lambda D \ . \ \lambda Y . \ #\{D \& Y\} > #D/2] =$

Extension[$\lambda D \ . \ \lambda Y . \ #\{D \& Y\} > #\{D \& \neg Y\}] =$

Extension[$\lambda D \ . \ \lambda Y . \ #\{D \& Y\} > #\{D\} - #\{D \& Y\}]$

(cp. Frege on Functions vs. Courses-of-Values, Marr on the function computed vs. the algorithm implemented)
Before E Chomsky (1986, ch. 1) on Languages

*i-language*: a *procedure* (intension) that connects “meanings” with “articulations” in a certain *constrained* way

Bingley is eager to please
Bingley is easy to please
Bingley is ready to please

(eager that he please others)
(easy for others to please him)
(ambiguous)
Before E. Chomsky (1986, ch. 1) on Languages

*i-language*: a procedure (intension) that connects “meanings” with “articulations” in a certain *constrained* way

*e-language*: a set (extension) of articulation-meaning pairs, or any another *nonprocedural* notion of language
Most of the dots are yellow

15 dots:
9 yellow
6 blue
‘Most of the dots are yellow’

**MOST**[DOT(x), YELLOW(x)]

\[\#\{x: DOT(x) \&\ YELLOW(x)\} > \#\{x: DOT(x)\}/2\]

More than half of the dots are yellow \( (9 > 15/2) \)

\[\#\{x: DOT(x) \&\ YELLOW(x)\} > \#\{x: DOT(x) \& \sim\YELLOW(x)\}\]

The yellow dots outnumber the non-yellow dots \( (9 > 6) \)

\[\#\{x: DOT(x) \&\ YELLOW(x)\} > \#\{x: DOT(x)\} – \#\{x: DOT(x) \&\ YELLOW(x)\}\]

The number of yellow dots exceeds the number of dots minus the number of yellow dots \( (9 > 15 – 9) \)
Hume’s Principle

\[ \# \{ x : T(x) \} = \# \{ x : H(x) \} \]

iff

\[ \{ x : T(x) \} \textit{ OneToOne } \{ x : H(x) \} \]

\[ \# \{ x : T(x) \} > \# \{ x : H(x) \} \]

iff

\[ \{ x : T(x) \} \textit{ OneToOnePlus } \{ x : H(x) \} \]

\[ \alpha \textit{ OneToOnePlus } \beta \text{ iff for some } \alpha^* , \]
\[ \alpha^* \text{ is a proper subset of } \alpha \text{, and } \alpha^* \textit{ OneToOne } \beta \]

(and it’s \textit{ not } the case that } \beta \textit{ OneToOne } \alpha)
‘Most of the dots are yellow’

\[\text{Most}[\text{DOT}(x), \text{YELLOW}(x)]\]

\[\#\{x:\text{DOT}(x) \land \text{YELLOW}(x)\} > \#\{x:\text{DOT}(x)\}/2\]

\[\#\{x:\text{DOT}(x) \land \text{YELLOW}(x)\} > \#\{x:\text{DOT}(x) \land \neg \text{YELLOW}(x)\}\]

\[\#\{x:\text{DOT}(x) \land \text{YELLOW}(x)\} > \#\{x:\text{DOT}(x)\} - \#\{x:\text{DOT}(x) \land \text{YELLOW}(x)\}\]

\[\text{{OneToOnePlus}}[\{x:\text{DOT}(x) \land \text{YELLOW}(x)\}, \{x:\text{DOT}(x) \land \neg \text{YELLOW}(x)\}]\]
‘Most of the dots are yellow’

\( \text{Most}^{[D, Y]} \)

\( \text{OneToOnePlus}[[D \land Y], [D \land \neg Y]] \)

\#\{D \land Y\} > \#\{D \land \neg Y\}

\#\{D \land Y\} > \#\{D\}/2

\#\{D \land Y\} > \#\{D\} - \#\{D \land Y\}

??Most of the paint is yellow???
Are most of the dots yellow?

What conditions make the question easy/hard to answer? That might provide clues about how the question is understood (given independent accounts of what information is available to humans in those conditions).
10 yellow, 8 blue
‘Most of the dots are yellow’

\[ \text{MOST}[D, Y] \]

\[ \text{OneToOnePlus}[[D \& Y], [D \& \sim Y]] \]

\[ \#D \& Y > \#D \& \sim Y \]

\[ \#D \& Y > \#D / 2 \]

\[ \#D \& Y > \#D - \#D \& Y \]
Some Relevant Facts

• many animals are good *cardinality-estimators*, by dint of a much studied system (see Dehaene, Gallistel/Gelman, etc.)
• appeal to *subtraction operations* is not crazy (Gallistel/King)
• infants can also do *one-to-one comparison* (see Wynn)
• Frege’s versions of the axioms for arithmetic can be derived (within a consistent fragment of Frege’s logic) from definitions and Hume’s *(one-to-one correspondence)* Principle

• *Lots of references in...*
  
  Interface Transparency and the Psychosemantics of ‘most’.
a model of the “Approximate Number System”
(key feature: *ratio-dependence* of discriminability)

distinguishing 8 dots from 4 (or 16 from 8)
is easier than
distinguishing 10 dots from 8 (or 20 from 10)
a model of the “Approximate Number System”
(key feature: ratio-dependence of discriminability)

correlatively, as the number of dots rises, “acuity” for estimating cardinality decreases--but still in a ratio-dependent way, with wider “normal spreads” centered on right answers
4:5 (blue:yellow)
“scattered pairs”
1:2 (blue:yellow)
“scattered pairs”
4:5 (blue:yellow)
“scattered pairs”
9:10 (blue:yellow)
“scattered pairs”
4:5 (blue:yellow)
“column pairs mixed”
5:4 (blue:yellow) "column pairs mixed"
4:5 (blue:yellow)
“column pairs sorted”
scattered random

scattered pairs

10 yellow, 8 blue (5:4 ratio)

column pairs mixed

column pairs sorted
Basic Design

• 12 naive adults, 360 trials for each participant
• 5-17 dots of each color on each trial
• trials varied by ratio (from 1:2 to 9:10) and type (scattered random/pairs, column pairs mixed/sorted)
• each “dot scene” displayed for 200ms
• target sentence: Are most of the dots yellow?
• answer ‘yes’ or ‘no’ by pressing buttons on a keyboard
• correct answer randomized
• controls for area (pixels) vs. number, yada yada...
better performance on easier ratios: $p < .001$
fits for Sorted-Columns trials to an independent model for detecting the longer of two line segments (apart from Sorted-Columns) to a standard psychophysical model for predicting ANS-driven performance.
performance on Scattered Pairs and Mixed Columns was no better than on Scattered Random; and it looks like the ANS was used to answer the question except in Sorted Columns trials (but it didn’t have to turn out that way)
scattered random

scattered pairs

10 yellow, 8 blue (5:4 ratio)

column pairs mixed
column pairs sorted
Side Point...50% plus a tad
Follow-Up Study

Could it be that speakers *analyze* ‘most’ in terms of a 1-To-1-Plus *concept*, but our task made it too hard to use a 1-To-1-Plus *verification strategy*, forcing subjects to use ancillary knowledge—e.g.,

\[
\text{1-To-1-Plus}[\{x: \text{Dot}(x) \& \text{Yellow}(x)\}, \{\text{Dot}(x) \& \sim\text{Yellow}(x)\}]
\]

iff

\[
\#\{x: \text{Dot}(x) \& \text{Yellow}(x)\} > \#\{\text{Dot}(x) \& \sim\text{Yellow}(x)\}
\]
Can subjects identify these trials, and (more importantly) do they notice the color of the “loners”?
better performance on components of a 1-to-1-plus task
fits for Sorted-Columns trials to an independent model for detecting the longer of two line segments

(apart from Sorted-Columns) to a standard psychophysical model for predicting ANS-driven performance
performance on Scattered Pairs and Mixed Columns was no better than on Scattered Random; looks like ANS was used to answer the question, except in the Sorted Columns trials.

<table>
<thead>
<tr>
<th>Trial Type</th>
<th>R²</th>
<th>Critical Weber Fraction</th>
<th>Nearest Whole-Number Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scattered Random</td>
<td>.9677</td>
<td>.32</td>
<td>3:4</td>
</tr>
<tr>
<td>Scattered Pairs</td>
<td>.8642</td>
<td>.33</td>
<td>3:4</td>
</tr>
<tr>
<td>Column Pairs Mixed</td>
<td>.9364</td>
<td>.30</td>
<td>3:4</td>
</tr>
<tr>
<td>Column Pairs Sorted</td>
<td>.9806</td>
<td>.04</td>
<td>25:26</td>
</tr>
</tbody>
</table>

Table 2. Parameter estimates from psychophysical model
‘Most of the dots are yellow’

\[
\text{MOST}^{[D, Y]} \\
\text{OneToOnePlus}[[D \& Y], [D \& \sim Y]] \\
\#\{D \& Y\} > \#\{D \& \sim Y\} \\
\#\{D \& Y\} > \#\{D\} - \#\{D \& Y\}
\]
‘Most of the dots are blue’

\[\#\{x: \text{Dot}(x) \& \text{Blue}(x)\} > \#\{x: \text{Dot}(x) \& \sim \text{Blue}(x)\}\]

\[\#\{x: \text{Dot}(x) \& \text{Blue}(x)\} > \#\{x: \text{Dot}(x)\} - \#\{x: \text{Dot}(x) \& \text{Blue}(x)\}\]

- if there are only two colors to worry about, say blue and red, then the \textit{non-blues} can be identified with the reds
‘Most of the dots are blue’

\[\{x: \text{Dot}(x) \& \text{Blue}(x)\} > \{x: \text{Dot}(x) \& \sim \text{Blue}(x)\}\]
\[\{x: \text{Dot}(x) \& \text{Blue}(x)\} > \{x: \text{Dot}(x)\} - \{x: \text{Dot}(x) \& \text{Blue}(x)\}\]

- if there are only two colors to worry about, say blue and red, then the non-blues can be identified with the reds
- the visual system can (and will) “select” the dots, the blue dots, and the red dots; so the ANS can (and will) estimate these three cardinalities

but adding more colors will make it harder (and with 5 colors, it will be impossible) for the visual system to make enough “selections” for the ANS to operate on
Figure 4

(a) 

(b) 

(c) 

(d)
‘Most’ as a Case Study

‘Most of the dots are blue’

\[\#\{x: \text{Dot}(x) \& \text{Blue}(x)\} > \#\{x: \text{Dot}(x) \& \sim\text{Blue}(x)\}\]
\[\#\{x: \text{Dot}(x) \& \text{Blue}(x)\} > \#\{x: \text{Dot}(x)\} - \#\{x: \text{Dot}(x) \& \text{Blue}(x)\}\]

• adding alternative colors will make it harder (and eventually impossible) for the visual system to make enough “selections” for the ANS to operate on

• so given the first proposal (with negation), verification should get harder as the number of colors increases

• but the second proposal (with subtraction) predicts relative indifference to the number of alternative colors
better performance on easier ratios: $p < .001$
no effect of number of colors
fit to psychophysical model of ANS-driven performance
Table 1. Parameter estimates from psychophysical model

<table>
<thead>
<tr>
<th>Trial Type</th>
<th>$R^2$</th>
<th>Critical Weber Fraction</th>
<th>Nearest Whole-Number Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Colors</td>
<td>.9480</td>
<td>.290</td>
<td>3:4</td>
</tr>
<tr>
<td>3-Colors</td>
<td>.9586</td>
<td>.320</td>
<td>3:4</td>
</tr>
<tr>
<td>4-Colors</td>
<td>.9813</td>
<td>.283</td>
<td>3:4</td>
</tr>
<tr>
<td>5-Colors</td>
<td>.9625</td>
<td>.316</td>
<td>3:4</td>
</tr>
</tbody>
</table>
‘Most’ as a Case Study

‘Most of the dots are blue’

\[ \#\{x: \text{Dot}(x) \land \text{Blue}(x)\} > \#\{x: \text{Dot}(x) \land \neg \text{Blue}(x)\} \]
\[ \#\{x: \text{Dot}(x) \land \text{Blue}(x)\} > \#\{x: \text{Dot}(x)\} - \#\{x: \text{Dot}(x) \land \text{Blue}(x)\} \]

• adding alternative colors will make it harder (and eventually impossible) for the visual system to make enough “selections” for the ANS to operate on

• so given the first proposal (with negation), verification should get harder as the number of colors increases

• but the second proposal (with subtraction) predicts relative indifference to the number of alternative colors
‘Most of the dots are yellow’

\[ \text{MOST}^t[D, Y] \]

OneToOnePlus[{D & Y}, {D & \sim Y}]

\[ \# \{D & Y\} > \# \{D & \sim Y\} \]

\[ \# \{D & Y\} > \# \{D\}/2 \]

\[ \# \{D & Y\} > \# \{D\} - \# \{D & Y\} \]

???Most of the paint is yellow???
‘Most’ as a Case Study

‘Most of the dots are blue’

\[\#\{x: \text{Dot}(x) \& \text{Blue}(x)\} > \#\{x: \text{Dot}(x)\} - \#\{x: \text{Dot}(x) \& \text{Blue}(x)\}\]

- mass/count flexibility

Most of the **dots (blobs) are** brown

Most of the **goo (blob) is** brown

- are mass nouns (somehow) disguised count nouns?

\[\#\{x: \text{GooUnits}(x) \& \text{BlueUnits}(x)\} > \#\{x: \text{GooUnits}(x)\} - \#\{x: \text{GooUnits}(x) \& \text{BlueUnits}(x)\}\]
discriminability is BETTER for ‘goo’ (than for ‘dots’)

$w = 0.18$

$r^2 = 0.97$

$w = 0.27$

$r^2 = 0.97$
Are more of the blobs blue or yellow?
   If more the blobs are blue, press ‘F’. If more of the blobs are yellow, press ‘J’.

Is more of the blob blue or yellow?
   If more the blob is blue, press ‘F’. If more of the blob is yellow, press ‘J’.
Performance is better (on the same stimuli) when the question is posed with a *mass noun*.
‘Most’ as a Case Study

‘Most of the dots are blue’
\[\#\{x:\text{Dot}(x) \& \text{Blue}(x)\} > \#\{x:\text{Dot}(x)\} - \#\{x:\text{Dot}(x) \& \text{Blue}(x)\}\]

- mass/count flexibility
  Most of the **dots (blobs)** are brown
  Most of the **goo (blob)** is brown

- are mass nouns disguised count nouns? SEEMS NOT
  \[\#\{x:\text{GooUnits}(x) \& \text{BlueUnits}(x)\} > \#\{x:\text{GooUnits}(x)\} - \#\{x:\text{GooUnits}(x) \& \text{BlueUnits}(x)\}\]
Are most of the peas blue?  Peas: more blue?
Is most if the corn blue?  Corn: more blue?
Procedure Matters

$\text{MOST}^*[D, Y]$

$\text{OneToOnePlus}[[D \& Y], [D \& \sim Y]]$

$\#D \& Y > \#D/2$

$\#D \& Y > \#D - \#D \& Y$

$\#D \& Y > \#D \& \sim Y$
But There’s Nothing Special About Success at Fast Speeds

• Failures at slow speeds can also be illuminating

• If you impose no time pressure and give people the information they need to answer a certain question, but they don’t settle on a judgment about the target sentence, then maybe people don’t understand the target sentence as posing that question.
The biggest circles are red
(pilot data: 100% yes)
The biggest circles are red
(pilot data: 85% yes)
The biggest circles are red

(pilot data: about 50% yes...compare)

There are some red circles that are bigger than all the other circles
The red lines are longer than the blue lines
(pilot data: about 0% say yes)
The red lines are longer than the blue lines
(pilot data: 30% say yes)
The red lines are longer than the other lines (pilot data: 100% say yes)
The red lines are longer than the other lines (pilot data: 60% say yes)
The red lines are longer than the other lines
(pilot data: 95% say yes)
The red lines are longer than the other lines
(pilot data: about 50% say yes)
The red lines are longer than the other lines (pilot data: 25% say yes)
The red lines are longer than the other lines
(pilot data: 0% say yes)
Old Idea: (1) understanding expressions is a matter of analyzing them in terms of “basic” concepts; so (3) understanding is importantly related to verification

- we don’t have a priori knowledge of when speakers will use a Logical Form as a Verification Procedure, as opposed to relying on ancillary beliefs and using an alternative procedure

- but reliable verification under time pressure can be illuminating, given good theories of which procedures can be used in that window

- unreliable verification with no time pressure can also be illuminating: it tells against a candidate Logical Form for a sentence S if speakers have trouble evaluating sentence S in situations that make it easy to evaluate the candidate Logical Form of S.
THANKS
...Psychophysics, on the other hand, is related more directly to the level of algorithm and representation. Different algorithms tend to fail in radically different ways as they are pushed to the limits of their performance or are deprived of critical information.

As we shall see, primarily psychophysical evidence proved to Poggio and myself that our first stereo-matching algorithm was not the one used by the brain, and the best evidence that our second algorithm (Marr and Poggio, 1976) is roughly the one used also comes from psychophysics. Of course, the underlying computational theory remained the same in both cases, only the algorithms were different. Psychophysics can also help to determine the nature of a representation...
“we can know the Markerese translation of an English sentence without knowing the first thing about the meaning of the English sentence; namely, the conditions under which it would be true. Semantics with no truth conditions is no semantics.”

Really? Is the first thing about the meaning of ‘The sky is blue’ or ‘John is eager to please’ the conditions under which the sentence would be true?

we can know the (alleged) conditions under which a sentence of a spoken language L would be true without knowing how (i.e., via what procedure) the sentence-sound is understood by any speakers of L. Semantics without understanding is no semantics.

We can stipulate that a semantics just is a T-semantics. But then the question is whether a human language has a semantics, as opposed to a U-semantics (cp. Harman, “Meaning and Semantics”)
Semantics: Truth or Understanding?

- **T-semantics: Tarski-style T-sentences as theorems**
  
  leave it open how a semantic theory for a human language H is related to the phenomena of understanding (or acquiring) H; don’t assume that a theory of H is theory of an i-language

- **U-semantics: a theory of understanding**
  
  leave it open how the theory is related to the natural phenomena of using expressions to make truth-evaluable claims; but assume that a theory of a human language is theory of an i-language

*Maybe* for each human language L, some good T-semantics for L will turn out to be a good U-semantics for L
I know of no promising way to make objective sense of the assertion that a computational procedure $\Gamma$ is used by a population $P$, whereas another procedure $\Gamma'$, which generates the same set of retinal-image/3D-sketch pairs as $\Gamma$, is not. I have tried to say how there are facts about $P$ which objectively select the Vision-sets used by $P$. I am not sure there are facts about $P$ which objectively select privileged computational procedures for those Vision-sets...

I think it makes sense to say that Vision-sets might be used by populations even if there were no internally represented procedures. I can tentatively agree that $V$ is used by $P$ if and only if everyone in $P$ possesses an internal representation of a procedure for $V$, if that is offered as a scientific hypothesis. But I cannot accept it as any sort of analysis of “$V$ is used by $P$”, since the analysandum clearly could be true although the analysans was false.
In the calculus of L-conversion and the calculus of restricted \( \lambda-K \)-conversion, as developed below, it is possible, if desired, to interpret the expressions of the calculus as denoting functions in extension.

However, in the calculus of \( \lambda-\delta \)-conversion, where the notion of identity of functions is introduced into the system by the symbol \( \delta \), it is necessary, in order to preserve the finitary character of the transformation rules, so to formulate these rules that an interpretation by functions in extension becomes impossible. The expressions which appear in the calculus of \( \lambda-\delta \)-conversion are interpretable as denoting functions in intension of an appropriate kind.”
Lewis, “Languages and Language”: E Before I

“What is a language? Something which assigns meanings to certain strings of types of sounds or marks. It could therefore be a function, a set of ordered pairs of strings and meanings.”

\[ |x - 1| + \sqrt{x^2 - 2x + 1} \quad \{(-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), \ldots\} \]
\[ \lambda x \cdot |x - 1| = \lambda x \cdot +\sqrt{x^2 - 2x + 1} \]

“What is language? A social phenomenon which is part of the natural history of human beings; a sphere of human action…”

“We may define a class of objects called grammars...
A grammar uniquely determines the language it generates. But a language does not uniquely determine the grammar that generates it.”
I know of no promising way to make objective sense of the assertion that a grammar \( \Gamma \) is used by a population \( P \), whereas another grammar \( \Gamma' \), which generates the same language as \( \Gamma \), is not. I have tried to say how there are facts about \( P \) which objectively select the languages used by \( P \). I am not sure there are facts about \( P \) which objectively select privileged grammars for those languages...a convention of truthfulness and trust in \( \Gamma \) will also be a convention of truthfulness and trust in \( \Gamma' \) whenever \( \Gamma \) and \( \Gamma' \) generate the same language.

I think it makes sense to say that languages might be used by populations even if there were no internally represented grammars. I can tentatively agree that £ is used by \( P \) if and only if everyone in \( P \) possesses an internal representation of a grammar for £, if that is offered as a scientific hypothesis. But I cannot accept it as any sort of analysis of “£ is used by \( P \)”, since the analysandum clearly could be true although the analysans was false.
Many Conceptions of Human Languages

- complexes of “dispositions to verbal behavior”
- strings of an elicited (or nonelicited) corpus
- something a radical interpreter ascribes to a speaker
- “Something which assigns meanings to certain **strings** of types of sounds or marks. It could therefore be a function, a **set** of ordered pairs of strings and meanings.”

- a **procedure that pairs** meanings with sounds/gestures