Mostly Framing

Paul M. Pietroski

University of Maryland

Dept. of Linguistics, Dept. of Philosophy
Outline

• Framing effects
  – may reveal laziness or puzzles
  – but also tell us something about *representational format*

• Some collaborative work on the meaning of ‘most’
  – review some published results, but framed in terms of framing
  – present some newer work, also framed this way

• As time permits, some collaborative work in progress on...
  – comparison: ‘more’ vs. ‘most’
  – plural superlatives: ‘The red circles are the biggest’
I Cognize, *ergo*
I am prone to Framing Effects

Examples via Kahneman’s recent book, *Thinking Fast and Slow*

A bat and a ball cost $1.10
The bat costs a dollar more than the ball

How much does the ball cost?

NOT ten cents...a dollar is not a dollar more than ten cents

Adam and Beth drive equal distances in a year.
Adam switches from a 12-mpg to 14-mpg car.
Beth switches from a 30-mpg to 40-mpg car.
Who will save more gas?

Adam: \(\frac{10,000}{12} = 833\) \(\frac{10,000}{14} = 714\) saving of 119 gallons
Beth: \(\frac{10,000}{30} = 333\) \(\frac{10,000}{40} = 250\) saving of 83 gallons
Schelling Effect

Suppose your tax depends on your income and how many kids you have.

- The “child deduction” might be flat, say 1000 per child
  \[ \text{Tax}(i, k) = \text{Base}(i) - [k \cdot 1000] \]
- Or it might depend on the taxpayer’s income
  \[ \text{Tax}(i, k) = \text{Base}(i) - [k \cdot \text{Deduction}(i)] \]

Q1: Should the child deduction be larger for the rich than for the poor?

Instead of taking the “standard” household to be childless, we could lower the base tax for every household (e.g., by 3000), and add a surcharge for households with less than 3 kids (e.g., 3000/2000/1000). We could also let the surcharge depend on income.

\[ \text{Tax}(i, k) = \text{LowerBase}(i) + [(3 - k) \cdot \text{Surcharge}(i)] \]

Q2: Should the childless poor pay as large a surcharge as the childless rich?
Schelling Effect

Q1: Should the child exemption be larger for the rich than for the poor?
Q2: Should the childless poor pay as large a surcharge as the childless rich?

if you answered ‘No’ to both, then you are not endorsing a coherent policy

*as Kahneman puts the point*...

the difference between the tax owed by a childless family and by a family with two children *can be described* as a *reduction* or as an *increase*

if you want the poor to receive at least the same *benefit* as the rich for having children, then you must want the poor to pay at least the same *penalty* as the rich for being childless.
1. \( \sim[Deduction(r) > Deduction(p)] \) \hspace{1cm} \text{Desire}

2. \( \text{Surcharge}(p) < \text{Surcharge}(r) \) \hspace{1cm} \text{Desire}

3. \text{for any income } i, \hspace{1cm} \text{obvious, but also provable}
   \( \text{Surcharge}(i) = Deduction(i) \)

4. \( \text{Surcharge}(r) = Deduction(r) \) \hspace{1cm} [3]

5. \( \text{Surcharge}(p) < Deduction(r) \) \hspace{1cm} [2, 4]

6. \( \text{Surcharge}(p) = Deduction(p) \) \hspace{1cm} [3]

7. \( \text{Deduction}(p) < Deduction(r) \) \hspace{1cm} [5, 6]

8. \( \text{Deduction}(r) > Deduction(p) \) \hspace{1cm} [7]

9. \( \bot \) \hspace{1cm} [1, 8]
Kahneman’s Conclusion

“The message about the nature of framing is stark: framing should not be viewed as an intervention that masks or distorts an underlying preference. At least in this instance...there is no underlying preference that is masked or distorted by the frame. Our preferences are about framed problems, and our moral intuitions are about descriptions, not substance.”

Perhaps things aren’t always this bad. But sometimes, robust “intuitions” reflect the format of mental representations, as opposed to some representation-neutral content. Theories that are (dis)confirmed by such intuitions may be theories of how—rather than what—the intuiters represent.
Most of the dots are yellow?

15 dots: 9 yellow 6 blue

How is the interrogative sentence understood? What question is getting asked?
‘Most of the dots are yellow’

\[ \text{MOST} \{\text{DOT, YELLOW}\} \]

15 dots: 9 yellow, 6 blue

\[ \#\{\text{DOT } \& \text{ YELLOW}\} > \#\{\text{DOT}\}/2 \]

More than half of the dots are yellow \( (9 > 15/2) \)

\[ \#\{\text{DOT } \& \text{ YELLOW}\} > \#\{\text{DOT } \& \sim \text{YELLOW}\} \]

The yellow dots outnumber the non-yellow dots \( (9 > 6) \)

\[ \#\{\text{DOT } \& \text{ YELLOW}\} > \#\{\text{DOT}\} \sim \#\{\text{DOT } \& \text{ YELLOW}\} \]

The number of yellow dots exceeds the number of dots minus the number of yellow dots \( (9 > 15 – 9) \)
Most of the dots are yellow?

\[
\text{MOST}[\text{DOT, YELLOW}]
\]

\[
\#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT}\}/2
\]

\[
\#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT} \& \sim\text{YELLOW}\}
\]

\[
\#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT}\} - \#\{\text{DOT} \& \text{YELLOW}\}
\]
Hume’s Principle

\#\{\text{Triangle}\} = \#\{\text{Heart}\}

iff

\{\text{Triangle}\} \text{ OneToOne } \{\text{Heart}\}

\-------------------------------\n
\#\{\text{Triangle}\} > \#\{\text{Heart}\}

iff

\{\text{Triangle}\} \text{ OneToOnePlus } \{\text{Heart}\}

\alpha \text{ OneToOnePlus } \beta \iff \text{ for some } \alpha^*,

\alpha^* \text{ is a proper subset of } \alpha, \text{ and } \alpha^* \text{ OneToOne } \beta

(and it’s not the case that } \beta \text{ OneToOne } \alpha)
Most of the dots are yellow

MOST[DOT, YELLOW]

OneToOnePlus[DOT & YELLOW, DOT & ~YELLOW]

#{DOT & YELLOW} > #{DOT}/2

#{DOT & YELLOW} > #{DOT & ~YELLOW}

#{DOT & YELLOW} > #{DOT} – #{DOT & YELLOW}
‘Most of the dots are yellow’

\[
\text{Most}[D, Y]
\]

OneToOnePlus[\{D \& Y\}, \{D \& \sim Y\}]

Number Representations

\#{D \& Y} > \#{D \& \sim Y}\]

\#{D \& Y} > \#{D}/2\]

\#{D \& Y} > \#{D} − \#{D \& Y}\]

These analyses are provably equivalent and not crazy
‘Most of the dots are yellow’

\[
\text{MOST}^{[D, Y]}\quad \Rightarrow \quad \#\{D \& Y\} > \#\{D\} - \#\{D \& Y\}
\]

??Most of the \textit{paint} is yellow??
‘Most of the dots are yellow’

$\textit{Most}^{[D, Y]}$

Why analyse at all?

Why not take ‘Most’ to be as primitive as ‘dot’ and ‘yellow’ seem to be?

Some of the yellow dogs barked $\Rightarrow$ Some of the dogs barked
Some of the dogs barked loudly $\Rightarrow$ Some of the dogs barked

None of the yellow dogs barked $\Leftarrow$ None of the dogs barked
None of the dogs barked loudly $\Leftarrow$ None of the dogs barked
‘Most of the dots are yellow’

\[ \text{MOST}^{[D, Y]} \]

Why analyse at all?
Why not take ‘Most’ to be as primitive as ‘dot’ and ‘yellow’ seem to be?

All of the yellow dogs barked \( \Leftarrow \) All of the dogs barked
All of the dogs barked loudly \( \Rightarrow \) All of the dogs barked

Most of the yellow dogs barked -- Most of the dogs barked
Most of the dogs barked loudly \( \Rightarrow \) Most of the dogs barked
‘Most of the dots are yellow’

\[
\text{MOST}^{[D, Y]}
\]

Why analyse at all?
Why not take ‘Most’ to be as primitive as ‘dot’ and ‘yellow’ seem to be?

Most of the dogs barked → More than half of the dogs barked
Most of the dogs barked → More dogs barked than didn’t

Most of the yellow dogs barked -- Most of the dogs barked
Most of the dogs barked loudly → Most of the dogs barked
Most of the dots are yellow?

\[ \text{MOST} [\text{DOT, YELLOW}] \]

\[ \text{OneToOnePlus} [\text{DOT & YELLOW, DOT & } \sim \text{YELLOW}] \]

\[ \#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT}\}/2 \]

\[ \#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT} \& \sim \text{YELLOW}\} \]

\[ \#\{\text{DOT} \& \text{YELLOW}\} > \#\{\text{DOT}\} - \#\{\text{DOT} \& \text{YELLOW}\} \]
Most of the dots are yellow?

What conditions make the interrogative easy/hard to answer?

That might provide clues about how the sentence is understood (given decent accounts of the information available to human beings in those conditions).
a model of the “Approximate Number System”
(key feature: \textit{ratio-dependence} of discriminability)

distinguishing 8 dots from 4 (or 16 from 8)
is easier than
distinguishing 10 dots from 8 (or 20 from 10)
a model of the “Approximate Number System”  
(key feature: *ratio-dependence* of discriminability)

correlatively, as the number of dots *rises*, “acuity” for estimating of cardinality  
decreases—-but still in a ratio-dependent way, with wider “normal spreads” centered on right answers
Most of the dots are yellow?

What conditions make the interrogative easy/hard to answer?

That might provide clues about how the sentence is understood (given decent accounts of the information available to human beings in those conditions).
4:5 (blue:yellow)
“scattered pairs”
1:2 (blue:yellow)
“scattered pairs”
4:5 (blue:yellow)
“scattered pairs”
9:10 (blue:yellow)
“scattered pairs”
4:5 (blue:yellow)
“column pairs sorted”
4:5 (blue:yellow)
“column pairs mixed”
5:4 (blue:yellow)
“column pairs mixed”
scattered random

scattered pairs

4:5 (blue:yellow)

column pairs mixed

column pairs sorted
Basic Design

- 12 naive adults, 360 trials for each participant
- 5-17 dots of each color on each trial
- trials varied by ratio (from 1:2 to 9:10) and type
- each “dot scene” displayed for 200ms
- target sentence: *Are most of the dots yellow?*
- answer ‘yes’ or ‘no’ by pressing buttons on a keyboard
- correct answer randomized
- controls for area (pixels) vs. number, etc.
4:5 (blue:yellow)

scattered random

scattered pairs

4:5 (blue:yellow)

THIS IS THE ONLY ODDBALL

column pairs mixed

column pairs sorted
better performance on easier ratios: $p < .001$
Table 2. Parameter estimates from psychophysical model

<table>
<thead>
<tr>
<th>Trial Type</th>
<th>$R^2$</th>
<th>Critical Weber Fraction</th>
<th>Nearest Whole-Number Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scattered Random</td>
<td>.9677</td>
<td>.32</td>
<td>3:4</td>
</tr>
<tr>
<td>Scattered Pairs</td>
<td>.8642</td>
<td>.33</td>
<td>3:4</td>
</tr>
<tr>
<td>Column Pairs Mixed</td>
<td>.9364</td>
<td>.30</td>
<td>3:4</td>
</tr>
<tr>
<td>Column Pairs Sorted</td>
<td>.9806</td>
<td>.04</td>
<td>25:26</td>
</tr>
</tbody>
</table>

Performance on Scattered Pairs and Mixed Columns was no better than on Scattered Random... looks like ANS was used to answer the question, except in Sorted Columns.
but **even better** performance on the components of a 1-to-1-plus task if the question is *not* posed with ‘most’
scattered random

scattered pairs

4:5 (blue:yellow)

column pairs mixed

column pairs sorted
‘Most of the dots are yellow’

\[
\text{MOST}[(D, Y)]
\]

\[
\text{OneToOnePlus}[[D \& Y], [D \& \sim Y]]
\]

Prima facie, posing the question with ‘most’ is not a way of posing a OneToOnePlus question

\[
\#(D \& Y) > \#(D \& \sim Y)
\]

\[
\#(D \& Y) > \#D - \#(D \& Y)
\]
‘Most of the dots are yellow’

\[
\text{\textit{Most}}^{[D, Y]}
\]

framing the question with ‘most’ has effects that are expected if the question is understood in terms of cardinality comparison (and answered via the ANS)

\[
\#\{D & Y\} > \#\{D & \sim Y\}
\]

\[
\#\{D & Y\} > \#\{D\} - \#\{D & Y\}
\]
Most of the dots are blue?

What conditions make the interrogative easy/hard to answer? That might provide clues about how the sentence is understood (given decent accounts of the information available to human beings in those conditions).
Most of the dots are blue?
Figure 4
‘Most of the dots are blue’

• #{Dot & Blue} > #{Dot & ~Blue}

  in scenes with two colors, blue and red, the non-blues can be identified with the reds

  #{Dot & ~Blue} = #{Dot & Red}

  the visual system will “select” the dots, the blue dots, and the red dots; these 3 sets will be estimated for cardinality

  but adding colors will make it harder (and with 5 colors, impossible) to obtain a cardinality estimate for each color

• #{Dot & Blue} > #{Dot} − #{Dot & Blue}
‘Most of the dots are blue’

- \#\{Dot & Blue\} > \#\{Dot & \neg\text{Blue}\}  
  
  verification should get \textbf{harder}  
  
  as the number of colors increases  

  “acuity” should be \textit{relatively good}  
  
  (w \approx .15)

- \#\{Dot & Blue\} > \#\{Dot\} − \#\{Dot & Blue\}  
  
  predicts \textit{indifference} to the number of alternative colors  

  “acuity” should be \textit{less good}  
  
  (w \approx .3)
‘Most of the dots are blue’

- \( \#\{\text{Dot} \& \text{Blue}\} > \#\{\text{Dot} \& \sim \text{Blue}\} \quad \text{15 dots, 9 blue} \)
  
  verification should get \textit{harder} 
  
  as the number of colors increases

  “acuity” should be \textit{relatively good} \quad (w \approx .15)

- \( \#\{\text{Dot} \& \text{Blue}\} > \#\{\text{Dot}\} - \#\{\text{Dot} \& \text{Blue}\} \quad \text{15 > (15 – 9)} \)
  
  predicts \textit{indifference} to the number of alternative colors

  “acuity” should be \textit{less good} \quad (w \approx .3)
better performance on easier ratios: $p < .001$
no effect of number of colors
fit to psychophysical model of ANS-driven performance
<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>2-Colors</td>
<td>.9480</td>
<td>.290</td>
<td>3:4</td>
</tr>
<tr>
<td>3-Colors</td>
<td>.9586</td>
<td>.320</td>
<td>3:4</td>
</tr>
<tr>
<td>4-Colors</td>
<td>.9813</td>
<td>.283</td>
<td>3:4</td>
</tr>
<tr>
<td>5-Colors</td>
<td>.9625</td>
<td>.316</td>
<td>3:4</td>
</tr>
</tbody>
</table>
‘Most of the dots are blue’

Prima facie, posing the question with ‘most’ is not a way of posing a cardinality question formulated in terms of negation.

\[
\text{MOST}[D, B] > \#{D} - \#{D \& B}
\]

\[
\#{D \& B} > \#{D \& \sim B}
\]
‘Most of the dots are blue’

$\text{MOST}^{[D, B]}$

framing the question with ‘most’ has effects that are expected if the question is understood in terms of cardinality $\text{subtraction}$

$\#\{D \& B\} > \#\{D\} - \#\{D \& B\}$

??Most of the $\text{paint}$ is blue??
‘Most of the dots are blue’

#{Dot & Blue} > #{Dot} – #{Dot & Blue}

• mass/count flexibility

Most of the *dots (blobs, peas, shoes) are* blue
Most of the *paint (blob, corn, footwear) is* blue

• are mass nouns (somehow) disguised count nouns?

#{GooUnit & BlueUnit} >
#{GooUnit} – #{GooUnit & BlueUnit}
First Study (blob/blobs...others in the works)

Performance is **better**, holding the scene constant, if the question is posed with a *mass noun*

$w = 0.20 \quad r^2 = 0.99$

$w = 0.29 \quad r^2 = 0.98$
‘Most of the dots are blue’
#{Dot & Blue} > #{Dot} – #{Dot & Blue}

• mass/count flexibility
Most of the *dots* *(blobs, peas, shoes)* *are* blue
Most of the *paint* *(blob, corn, footwear)* *is* blue

• are mass nouns (somehow) disguised count nouns?
#{GooUnit & BlueUnit} >  
#{GooUnit} – #{GooUnit & BlueUnit} SEEMS NOT
Most of the dots are blue?

\[ \text{MOST}^{|D, B|} \]

framing the question with ‘most’ has effects that are expected if the question is understood in terms of cardinality **subtraction**

Prima facie, this requires a representation of \( \# \{D\} \) and a computation on this representation.

\[ \# \{D \& B\} > \# \{D\} - \# \{D \& B\} \]

Does use of ‘most’ reflect ease/difficulty of representing \( \# \{D\} \)?
Of Undergrads Who Choose This Side (N=48)

Asked, “Most” = 58%

Asked, “More” = 13%
Which sentence would you choose to describe this picture?

A) More of the dots are grey.

B) Most of the dots are grey.
Which sentence would you choose to describe this picture?

A) More of the dots are grey.

B) Most of the dots are grey.
Not just about recognition

• 80 participants asked to “draw” on an iPad

*More/Most of the dots are blue*
Not just about recognition

• Typical for “More of the dots are blue”
Not just about recognition

• Typical for “Most of the dots are blue”
Centroid distance (adults)

More

Most

4-8 yr olds (n=92)

More

Most
Future Directions

• While successes at 200ms can be illuminating, our claim is not that 200ms is the “understanding window.” But given the information available, different claims about how sentences are understood may lead to different predictions about how well competent speakers can evaluate the sentences.

• Failures with no time constraints can also be illuminating (recall the Schelling puzzle about tax deductions)

• If people have the information needed to answer question Q, but they don’t settle on judgments about a target sentence, then maybe the target sentence does not pose question Q.
The red circles are bigger than the other circles
The red circles are the biggest
The biggest circles are red

(pilot data: 100% yes)
The red circles are bigger than the other circles
The red circles are the biggest
The biggest circles are red

(pilot data: 85% yes)
Could it be that mixing plurality and comparison doesn’t determine a clear truth condition?

The red circles are bigger than the other circles
The red circles are the biggest
The biggest circles are red

(pilot data: about 50% yes...doesn’t look like 2 subgroups)

what are the meanings such that the sentences and the scene *don’t* determine the answers?
The red lines are longer than the other lines (pilot data: 100% say yes)
The red lines are longer than the other lines
(pilot data: 60% say yes)
The red lines are longer than the blue lines
(pilot data: about 0% say yes)
The red lines are longer than the blue lines
(pilot data: 30% say yes)
The red lines are longer than the other lines
(pilot data: 95% say yes)
The red lines are longer than the other lines (pilot data: about 50% say yes)
The red lines are longer than the other lines
(pilot data: 25% say yes)
potential puzzles for *any* theory according to which the sentences have compositionally determined representation-neutral conditions

The red lines are longer than the other lines (pilot data: 0% say yes)
Mostly Framing
Paul M. Pietroski
University of Maryland
Dept. of Linguistics, Dept. of Philosophy
Torcello was moved to Venice.
Venice is a nice place.
Venice may need to be moved.

Torcello was moved to a nice place that may need to be moved.