(1) [5] Give a general solution of the equation

\[(D^2 + 8D + 20)^2(D - 5)^3 y = 0, \quad \text{where} \quad D = \frac{d}{dt}.\]

**Solution.** This is a seventh-order, homogeneous, linear equation that has constant coefficients. Its characteristic polynomial is

\[p(z) = (z^2 + 8z + 20)^2(z - 5)^3 = \left(\left(z + 4 \right)^2 + 2^2 \right)^2(z - 5)^3,\]

which has roots \(-4 + i2, -4 + i2, -4 - i2, -4 - i2, 5, 5, \text{ and } 5\). A general solution of the differential equation is

\[y(t) = c_1 e^{-4t} \cos(2t) + c_2 e^{-4t} \sin(2t) + c_3 t e^{-4t} \cos(2t) + c_4 t e^{-4t} \sin(2t) + c_5 e^{5t} + c_6 t e^{5t} + c_7 t^2 e^{5t}.\]

The reasoning is as follows:

- the double conjugate pair \(-4 \pm i2\) yields the solutions \(e^{-4t} \cos(2t), e^{-4t} \sin(2t), t e^{-4t} \cos(2t), \text{ and } t e^{-4t} \sin(2t)\);

- the triple real root 5 yields the solutions \(e^{5t}, t e^{5t}, \text{ and } t^2 e^{5t};\)

(2) [3] Give the degree, characteristic, and multiplicity for the forcing term of the equation

\[y'' - 8y' + 25y = 7t^5 e^{4t} \sin(3t).\]

**Solution.** This is a second-order, nonhomogeneous, linear equation with constant coefficients. Its characteristic polynomial is \(p(z) = z^2 - 8z + 25 = (z - 4)^2 + 3^2\), which has roots \(4 \pm i3\).

The forcing term \(7t^5 e^{4t} \sin(3t)\) has degree \(d = 5\), characteristic \(\mu + iv = 4 + i3\), and multiplicity \(m = 1\).

(3) [2] Give a particular solution of the equation

\[y'' - 10y' + 25y = 8e^{3t}.\]

**Solution.** This is a second-order, nonhomogeneous, linear equation with constant coefficients. Its characteristic polynomial is \(p(z) = z^2 - 10z + 25 = (z - 5)^2\), which has double root 5. Its forcing has degree \(d = 0\), characteristic \(\mu + iv = 3\), and multiplicity \(m = 0\).

**Key Identity Evaluations.** Because \(m + d = 0\) we need only the Key Identity

\[L(e^{zt}) = (z^2 - 10z + 25)e^{zt}.\]

By evaluating the Key Identity at the characteristic \(z = 3\) we obtain

\[L(e^{3t}) = (3^2 - 10 \cdot 3 + 25)e^{3t} = (9 - 30 + 25)e^{3t} = 4e^{3t}.\]

Upon multiplying this by 2 we see that a particular solution is \(y_P(t) = 2e^{3t}\).