Quiz 10 Solutions, Math 246, Professor David Levermore
Tuesday, 1 December 2015

(1) [5] A $2 \times 2$ matrix $A$ has the eigenpairs
\[
\left(1, \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right), \quad \left(2, \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right).
\]
Sketch a phase-plane portrait that indicates typical orbits for the system $x' = Ax$. Identify its type. Classify the origin as either attracting, stable but not attracting, unstable but not repelling, or repelling.

**Solution.** Because $A$ has two positive eigenvalues, the phase portrait is a *nodal source*. The origin is thereby *repelling*. The phase portrait should show one orbit that moves away from the origin along each half of the lines $y = \frac{1}{2}x$ and $y = -x$. The phase portrait should indicate that every other orbit emerges from the origin tangent to the line $y = \frac{1}{2}x$.

(2) [5] Sketch a phase-plane portrait that indicates typical orbits for the system
\[
\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.
\]
Identify its type. Classify the origin as either attracting, stable but not attracting, unstable but not repelling, or repelling.

**Solution.** The characteristic polynomial of the given matrix $A$ is
\[
p(z) = z^2 - \text{tr}(A)z + \det(A) = z^2 + 6z + 13 = (z + 3)^2 + 4.
\]
We see that the mean $\mu = -3$ and the discriminant $\delta = -4$. Because $\delta = -4 < 0$, there are no real eigenpairs. Because $\mu = -3 < 0$, $\delta = -4 < 0$, and $a_{21} = -1 < 0$ the phase portrait is a *clockwise spiral sink*. The origin is thereby *attracting*. The phase portrait should indicate a family of clockwise spiral orbits that approach the origin.