Phase stability, electrochemical stability and ionic conductivity of the Li$_{10\pm\text{1}}$MP$_2X_{12}$ ($M = \text{Ge, Si, Sn, Al or P,}$ and $X = O, S$ or Se) family of superionic conductors†

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We present an investigation of the phase stability, electrochemical stability and Li$^+$ conductivity of the Li$_{10\pm\text{1}}$MP$_2X_{12}$ ($M = \text{Ge, Si, Sn, Al or P,}$ and $X = O, S$ or Se) family of superionic conductors using first principles calculations. The Li$_{10}$GeP$_2S_{12}$ (LGPS) superionic conductor has the highest Li$^+$ conductivity reported to date, with excellent electrochemical performance demonstrated in a Li-ion rechargeable battery. Our results show that isovalent cation substitutions of Ge$^{4+}$ have a small effect on the relevant intrinsic properties, with Li$_{10}$SiP$_2S_{12}$ and Li$_{10}$SnP$_2S_{12}$ having similar phase stability, electrochemical stability and Li$^+$ conductivity as LGPS. Alkaline cation substitutions ($M = \text{Al or P}$) with compensating changes in the Li$^+$ concentration also have a small effect on the Li$^+$ conductivity in this structure. Anion substitutions, however, have a much larger effect on these properties. The oxygen-substituted Li$_{10}$MP$_2$O$_{12}$ compounds are predicted not to be stable (with equilibrium decomposition energies $>90$ meV per atom) and have much lower Li$^+$ conductivities than their sulfide counterparts. The selenium-substituted Li$_{10}$MP$_2$Se$_{12}$ compounds, on the other hand, show a marginal improvement in conductivity, but at the expense of reduced electrochemical stability. We also studied the effect of lattice parameter changes on the Li$^+$ conductivity and found the same asymmetry in behavior between increases and decreases in the lattice parameters, i.e., decreases in the lattice parameters lower the Li$^+$ conductivity significantly, while increases in the lattice parameters increase the Li$^+$ conductivity only marginally. Based on these results, we conclude that the size of the $S^{2-}$ is near optimal for Li$^+$ conduction in this structural framework.

**Broader context**

Solid lithium superionic conductors exhibit good safety and stability, and are promising to replace organic liquid electrolytes in lithium-ion batteries. The current state-of-the-art Li$_{10}$GeP$_2S_{12}$ superionic conductor has the highest conductivity ever achieved among the solid lithium electrolytes of $12 \text{ mS cm}^{-1}$ at room temperature and excellent electrochemical performance. In this work, we attempt to address two key limitations of Li$_{10}$GeP$_2S_{12}$, namely the high cost of germanium and its air and moisture sensitive sulfide-based chemistry, by studying the phase stability, electrochemical stability and Li$^+$ conductivity of the Li$_{10\pm\text{1}}$MP$_2X_{12}$ family of superionic conductors ($M = \text{Ge, Si, Sn, Al or P,}$ and $X = O, S$ or Se) using first principles calculations. We find that the first limitation can be addressed by substituting relatively cheap silicon or tin for germanium in Li$_{10}$GeP$_2S_{12}$, with limited impact on stability and conductivity. However, the second limitation cannot be easily addressed by substituting oxide-based chemistry for sulfide-based chemistry; Li$_{10}$MP$_2$O$_{12}$ compounds are predicted to be significantly less stable and have lower conductivity than their sulfide counterparts. We also quantify the effect of ion substitutions, lattice parameter changes and Li concentrations on various electrochemical properties, which provides a useful insight into the optimization of LGPS and other similar materials.

**Introduction**

The continued drive for high performance lithium-ion batteries has imposed stricter requirements on the electrolyte materials.¹

Solid electrolytes comprising lithium superionic conductor materials exhibit good safety and stability, and are promising to replace current organic liquid electrolytes.²–⁵ However, one major limitation in the application of Li-ion conductors is that their typical conductivity is less than $10^{-4} \text{ S cm}^{-1}$ at room temperature.

In September 2011, Kamaya et al. reported a new Li superionic conductor Li$_{10}$GeP$_2S_{12}$ (LGPS), which has the highest conductivity ever achieved among the solid lithium electrolytes of $12 \text{ mS cm}^{-1}$ at room temperature (comparable conductivity with liquid electrolytes), and outstanding electrochemical performance in Li-ion batteries.⁶ The high conductivity in LGPS...
is attributed to the fast diffusion of Li\(^+\) in its crystal structural framework (see Fig. 1), which consists of \((\text{Ge}_{0.5}\text{P}_{0.5})\text{S}_4\) tetrahedra, \(\text{PS}_4\) tetrahedra, \(\text{LiS}_6\) octahedra, and \(\text{LiS}_4\) tetrahedra. Kamaya \textit{et al.} proposed that diffusion in LGPS occurs along one-dimensional (1D) pathways along the \(c\)-axis.\(^6\) The authors also proposed that Li atoms in \(\text{LiS}_6\) tetrahedra enable fast diffusion along the \(c\)-direction, while Li atoms in \(\text{LiS}_6\) octahedra are not active for diffusion. This hypothetical diffusion mechanism in LGPS has been inferred from the large anisotropic thermal factors and the Li disorder in the 1D channels.

Earlier, we investigated the phase stability, electrochemical stability and Li\(^+\) conductivity of LGPS using first principles techniques.\(^7\) We find that LGPS is a metastable phase in the calculated Li–Ge–P–S phase diagram. We also find that LGPS is not stable against reduction by lithium at low voltage or extraction of Li with decomposition at high voltage. Together with the calculated band gap of 3.6 eV, these predictions suggest that the observed electrochemical window of >5 V for this material is likely the result of a passivation phenomenon, where either \(\text{LiS}_6\) or \(\text{PS}_4\) is formed as a decomposition product. Furthermore, while \textit{ab initio} molecular dynamics (MD) simulations confirm fast Li\(^+\) diffusion in the 1D diffusion channel along the \(c\)-direction, they also predict two additional diffusion pathways in the \(a\)-\(b\) plane. Though diffusion in the \(a\)-\(b\) plane is not as facile as in the \(c\)-direction, it nonetheless contributes to the overall performance of the material. A later work by Adams and Prasada Rao\(^8\) using classical MD simulations with a force field based on Morse-type interactions derived from bond valence parameters similarly found weakly anisotropic diffusion in LGPS. In practice, diffusion along more than one dimension is necessary to obtain Li transport over reasonable distances.\(^9\)

Despite its ground-breaking Li\(^+\) conductivity and excellent electrochemical performance, LGPS still suffers from two significant obstacles to its adoption as a solid-electrolyte material. First, there is a practical matter of the use of relatively rare and expensive germanium in LGPS, which would limit large-scale application of the material. Second, sulfide-based compounds tend to be air and moisture sensitive, which could pose problems for cost-effective synthesis and application. These limitations of LGPS motivate us to look for other compounds in the same structural framework that can achieve a better balance between electrochemical performance, cost and other properties.

![Fig. 1](image.png)

\textbf{Fig. 1} Crystal structure of \(\text{Li}_{10}\text{GeP}_2\text{S}_{12}\). Large yellow atoms: S; small green atoms: fully occupied Li sites; small green-white atoms: partially occupied Li sites; red tetrahedra: \((\text{Ge}_{0.5}\text{P}_{0.5})\text{S}_4\); and blue tetrahedra: \(\text{PS}_4\).

In this work, we investigate the phase stability, electrochemical stability and Li\(^+\) conductivity of the \(\text{Li}_{10}\text{GeP}_2\text{S}_{12}\) (LMPX) family of superionic conductors, where \(M = \text{Ge}, \text{Si}, \text{Sn}, \text{Al} \text{or P}, \text{and } X = \text{O}, \text{S} \text{or Se}\), using first principles calculations. We seek to elucidate the effect of cation and anion substitutions on these three key properties. We include aliovalent cation substitutions to study the effect of the Li\(^+\) concentration on Li\(^+\) conductivity. We also study the effect of lattice parameters on Li\(^+\) conductivity in the LGPS structural framework.

**Methods**

All calculations in this work were performed using the Vienna \textit{Ab initio} Simulation Package (VASP)\(^10\) within the projector augmented-wave approach.\(^11\) Given the vastly different requirements of the various techniques used in this paper, we have carefully selected the appropriate functionals and methods for each technique based on accuracy and computational cost considerations, as outlined in the following sections.

**Phase stability**

The phase stability of the various LMPX structures was investigated by constructing the relevant Li–M–P–X computational phase diagrams.\(^12,13\) To ensure a good coverage of the phase space, we not only included all known Li–M–P–X compounds in the Inorganic Crystal Structure Database,\(^14\) but also included structures derived from the following sources:

1. All Li\(_2\text{P}_2\text{S}_4\) compounds compiled by Holzwarth \textit{et al.}\(^15\)

2. All possible inter-substitutions of existing Li–M–X and Li–P–X compounds. For example, the Li–P–O phase space is generally more well-studied than the Li–P–S and Li–P–Se phase space. In fact, some of the Li\(_2\text{P}_2\text{S}_4\) compounds compiled by Holzwarth \textit{et al.}\(^15\) have been obtained by substituting O for S in Li\(_2\text{P}_2\text{O}_4\) compounds. We adopted a similar strategy to ensure a good coverage of all the investigated phase spaces, e.g., we performed O for Se and S for Se substitutions to obtain possible Li–P–Se and Li–M–Se phases.

All total energy calculations for phase stability analysis were performed using the Perdew–Burke–Ernzerhof (PBE) generalized-gradient approximation (GGA)\(^16\) functional. A \(k\)-point density of at least 500/(number of atoms in the unit cell) was used for all calculations. All calculations were spin-polarized.

As the refined LGPS structure (\(P4_3/nmc\)) has partial occupancies on Li sites as well as Ge/P sites (see Fig. 1),\(^4\) we ordered the arrangement of Li, Ge, and P atoms in LGPS using an electrostatic energy criterion\(^17\) using the Python Materials Genomics (pymatgen) analysis code.\(^18\) All ions were assigned “idealized” charges based on their valence states, i.e., +1 for Li, +4 for Ge, +5 for P and –2 for S. We then selected 30 structures with the lowest electrostatic energy and relaxed them using density functional theory (DFT). It should be noted that the structure that minimizes the electrostatic energy is not the lowest energy structure based on DFT calculations. While the lowest electrostatic energy ordered structure still has a tetragonal \(P4_3/mc\) space group, the lowest DFT energy structure has a \(P1\) space group with a unit cell that is slightly distorted from the...
tetragonal cell (see the ESI† for details). The DFT energy of the relaxed $P1$ structure is approximately 10 meV per atom lower than the DFT energy of the structure that minimizes the electrostatic energy for all the LMPX compounds investigated. Henceforth, we will use the DFT ground state structure of LGPS for the remainder of our analyses. We note that while it is possible that there exist orderings (either for a single unit cell or at larger super cell sizes) that could have lower DFT energies, they are unlikely to be significantly lower in energy. Furthermore, the comparison of phase stability is carried out between materials with similar orderings, and we expect relative phase stability to be well characterized even if the exact lowest energy ordering is not obtained. At temperatures of interest (e.g., room-temperature), the Li⁺ sites are likely to be disordered, and hence, the structure would have higher symmetry.

All derived isovalent LMPX structures are based on M for Ge and X for S substitution of the DFT ground state structure. For isovalently substituted structures [Al³⁺ or P⁵⁺ for Ge³⁺ with corresponding changes in the Li⁺ concentration], we performed a similar electrostatic ordering of the structure and then performed DFT calculations on the lowest electrostatic energy structure only, i.e., we did not perform DFT calculations on multiple possible orderings, given the significant computational time involved. Only the sulfide versions of the Al and P structures are investigated in this work. Please see the ESI† for details on the relaxed structures.

Electrochemical stability

We assessed the electrochemical stability of the LMPX compounds using two approaches:

(1) Intrinsic stability against inert electrodes. We assessed the intrinsic stability of the LMPX solid electrolyte with respect to inert electrodes by calculating the band gap of the material.⁶⁹ As standard semi-local DFT is known to severely underestimate band gaps, the density of states (DOS) of all LMPX compounds were calculated using the Heyd-Scuseria-Emzerhof (HSE) screened hybrid functional,⁶⁸,⁶⁹ which has been tested to give relatively accurate band gaps for a wide range of materials.⁶⁸,⁶⁹ Owing to the relatively high computational expense of HSE over PBE, non-spin-polarized calculations were performed. The bandgap itself is not an exact measure of the electrochemical stability on inert electrodes as its alignment with respect to an external reference potential is not known. It can, however, be considered as an upper bound for the electrochemical window.

(2) Chemical stability against electrodes. We also investigated the chemical stability of the LMPX solid electrolyte against typical electrode materials used in lithium-ion batteries. We constructed the lithium grand potential Li–M–P–X phase diagrams using the method outlined by Ong et al.¹²,¹³ Lithium grand potential phase diagrams represent the phase equilibria of a Li–M–P–X system that is open to lithium, which is relevant when the LMPX solid electrolyte is in contact with a reservoir or sink of lithium, as is the case in a lithium battery. The voltage in a battery is the negative of the Li chemical potential. By studying the phase evolution of the Li₇₀₋ₓMₓPₓX₂ composition with respect to changing lithium chemical potential ($\mu_{\text{Li}}$), we can determine the equilibrium phases at $\mu_{\text{Li}}^0$ corresponding to the bulk metallic lithium (anode) and ($\mu_{\text{Li}}^0 - 5$) eV corresponding to a 5 V charged cathode. The equilibrium phases provide insight into how sensitive the Li₁₀₋ₓMₓPₓX₂ composition is to lithium insertion or extraction at high and low voltages and whether the phases formed would affect lithium conductivity. We should note that this analysis pertains to only one possible reaction path with the electrodes, i.e., the exchange of Li between the electrolyte and the electrode. In the case of the cathode, other possible reactions involving non-Li species in the cathode material were not investigated.

Li⁺ diffusivity and conductivity

We investigated the Li⁺ diffusivity and conductivity in the LMPX materials using ab initio molecular dynamics (AIMD) simulations. The AIMD simulations were performed using the PBE GGA functional.⁶⁸ To keep the computational cost at a reasonable level, smaller plane wave energy cut-offs of 400 eV, 280 eV and 270 eV were chosen as for oxides, sulfides, and selenides, respectively. A minimal Γ-centered $1 \times 1 \times 1$ k-point grid was used, and all calculations were non-spin-polarized.

The AIMD simulations were performed on one unit cell of LMPX. We performed convergence tests on a $2 \times 2 \times 1$ supercell of the original LGPS structure and found that a single unit cell is sufficient to obtain converged diffusivity and conductivity numbers. The volume of the unit cell and the initial position of atoms were obtained from the fully relaxed cells in the phase stability calculations. The integration of Newton’s equation is based on the Verlet algorithm implemented in VASP. The time step of molecular dynamics was chosen to be 2 fs. The procedure of the AIMD simulations is as follows:

1. At the start of the MD simulations, the LMPX samples are assigned an initial temperature of 100 K according to a Boltzmann distribution.

2. The samples are then heated up to the desired temperature (600 to 1200 K) by velocity scaling over 1000 time steps (2 ps), and then equilibrated at the equilibrium temperature for 5000 time steps (10 ps) in the NVT ensemble with a constant volume and with a Nosé–Hoover thermostat.²⁴,²⁵

3. The MD simulations for diffusion are then performed for 40 ps to 400 ps in the NVT ensemble until the diffusion coefficient is converged. We exclude data points where melting or breaking of M–X bonds is observed.

The diffusion coefficient is defined as the mean square displacement over time:

$$D = \frac{1}{2d} \left\langle |\mathbf{r}(t)|^2 \right\rangle,$$

where $d$ equals to 3, which is the dimension of the lattice in which diffusion takes place. The average mean square displacement $\left\langle |\mathbf{r}(t)|^2 \right\rangle$ was calculated as:

$$\left\langle |\mathbf{r}(t)|^2 \right\rangle = \frac{1}{N} \sum_{i} \left\langle |\mathbf{r}_i(t + \Delta t) - \mathbf{r}_i(t)|^2 \right\rangle,$$

where $\mathbf{r}_i(t)$ is the displacement of the $i$-th Li ion at time $t$. The calculated displacement $\mathbf{r}_i(t)$ is the displacement of an individual Li atom.
Table 1  Phase equilibria and decomposition energies for Li_{10+ε}MP_{2}X_{12}

<table>
<thead>
<tr>
<th>Cation (M)</th>
<th>Anion (X)</th>
<th>Phase equilibria at Li_{10+ε}MP_{2}X_{12} composition</th>
<th>( E_{\text{decomp}} ) (meV per atom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>O</td>
<td>Li_{2}SiO_{4} + 2Li_{1}PO_{4}</td>
<td>92</td>
</tr>
<tr>
<td>Ge</td>
<td>O</td>
<td>Li_{2}GeO_{4} + 2Li_{1}PO_{4}</td>
<td>96</td>
</tr>
<tr>
<td>Sn</td>
<td>O</td>
<td>0.33Li_{2}SnO_{4} + 0.67Li_{1}SnO_{4} + 2Li_{1}PO_{4}</td>
<td>97</td>
</tr>
<tr>
<td>Si</td>
<td>S</td>
<td>Li_{2}SiS_{4} + 2Li_{1}PS_{4}</td>
<td>19</td>
</tr>
<tr>
<td>Ge</td>
<td>S</td>
<td>Li_{2}GeS_{4} + 2Li_{1}PS_{4}</td>
<td>25</td>
</tr>
<tr>
<td>Sn</td>
<td>S</td>
<td>Li_{2}SnS_{4} + 2Li_{1}PS_{4}</td>
<td>25</td>
</tr>
<tr>
<td>Al</td>
<td>S</td>
<td>Li_{2}AlS_{4} + 2Li_{1}PS_{4}</td>
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</tr>
<tr>
<td>P</td>
<td>S</td>
<td>3Li_{1}PS_{4}</td>
<td>22</td>
</tr>
<tr>
<td>Si</td>
<td>Se</td>
<td>Li_{2}SiSe_{4} + Li_{1}PSe_{6} + Li_{1}Se + Se</td>
<td>16</td>
</tr>
<tr>
<td>Ge</td>
<td>Se</td>
<td>Li_{2}GeSe_{4} + Li_{1}PSe_{6} + Li_{1}Se + Se</td>
<td>16</td>
</tr>
<tr>
<td>Sn</td>
<td>Se</td>
<td>Li_{2}SnSe_{4} + Li_{1}PSe_{6} + Li_{1}Se + Se</td>
<td>19</td>
</tr>
</tbody>
</table>

The average mean square displacement is an average over \( N \) of all Li ions and is an ensemble average over time \( t_{0} \). Therefore, the calculated diffusion coefficient \( D \) is the self-diffusion of Li\(^{+} \) ions rather than the combined diffusion of the center of mass of all Li\(^{+} \) ions. It is known that these two definitions of diffusion coefficients become equivalent if there is no cross-correlation between displacement \( r(t) \) of different particles at different times.\(^{28} \) The value of \( D \) is obtained by performing a linear fitting to the relationship of average mean square displacement versus \( 2dt \).

Results

Stability and relaxed structural parameters

Table 1 shows the thermodynamic phase equilibria determined for a Li_{10+ε}MP_{2}X_{12} (M = Ge, Si, Sn, Al or P and X = O, S or Se) composition in the Li–M–P–X quaternary phase diagram, as well as the calculated equilibrium decomposition energies (rightmost column). The calculated equilibrium decomposition energy \( E_{\text{decomp}} \) is a measure of the stability of a material and is defined as the negative of the reaction energy per atom for the Li_{10+ε}MP_{2}X_{12} compound to decompose to the predicted thermodynamic equilibrium mixture of stable phases for that composition. Stable materials have an \( E_{\text{decomp}} \) of 0, and the higher the \( E_{\text{decomp}} \) the less likely a structure would be stable. All stable phases in the Li–M–P–X systems investigated are tabulated in the ESI.\(^{†} \)

All Li_{10+ε}MP_{2}X_{12} compounds are found to be thermodynamically unstable in our calculations. From Table 1, we may identify several clear trends in the stability of the various cation/anion-substituted Li_{10+ε}MP_{2}X_{12} structures. In general, we find that the oxides to be highly unstable in this structure, with \( E_{\text{decomp}} > 90 \) meV per atom. The sulfide and selenide structures have somewhat similar stability of less than 25 meV per atom, which are sufficiently small that these compounds can be easily stabilized by entropic effects or created as metastable phases. With the exception of the aliovalently substituted Li_{11}AlP_{2}S_{12}, the cation does not seem to significantly affect the stability of the Li_{10+ε}MP_{2}X_{12} structure. It may be observed that for the oxides and sulfides in general, the equilibrium phases comprise Li_{1}MX_{4} + Li_{1}PX_{4}. The only exception is Li_{10}SnP_{2}O_{12} for which Li_{1}SnO_{4} is unstable versus Li_{8}SnO_{6} and Li_{2}SnO_{3}. For the selenides, the equilibrium breakdown also contains Li_{1}MSe_{4}, but Li_{1}PSe_{4} is unstable against a combination of Li_{1}PSe_{6} + Li_{1}Se + Se. Li_{1}PSe_{6} is a Se-substituted version of the known Li_{1}PSe_{6} compound.\(^{27} \) We also note that the synthesis of Li_{1}SnS_{4} (isostructural with Li_{1}GeS_{4}) has only been reported recently\(^{28} \) and is indeed predicted to be stable by our calculations.

Table 2 shows the relaxed structural parameters for the Li_{10}MP_{2}X_{12} compounds investigated. We may observe that the unit cell volumes of Li_{10}MP_{2}O_{12} compounds are around 53–55% smaller than the corresponding Li_{10}MP_{2}S_{12} compounds, while the Li_{10}MP_{2}Se_{12} compounds have unit cell volumes that are around 16–19% larger than the corresponding Li_{10}MP_{2}S_{12} compounds. A significant proportion of the differences in volume is accounted for by the differences in the anionic radii. The ionic radii of O\(^{2–} \), S\(^{2–} \) and Se\(^{2–} \) are 126, 170 and 184 pm respectively,\(^{29} \) which gives \( \frac{V_{O}}{V_{S}} = \left( \frac{r_{O}^{2}}{r_{S}^{2}} \right)^{3} = 0.40 \) and \( \frac{V_{Se}}{V_{S}} = \left( \frac{r_{Se}^{2}}{r_{S}^{2}} \right)^{3} = 1.26. \)

We also performed a topological analysis of the relaxed Li_{10}MP_{2}X_{12} compounds using the open source Zeo++ software.\(^{30,31} \) For all materials, we removed all Li in the structure and then calculated the largest free sphere that can pass through the structural framework by the remaining cations and anions, which is designated as the “channel size” in Table 2. As we can see, the oxide structures tend to have channel sizes that are around 20% smaller compared to the sulfides,

<table>
<thead>
<tr>
<th>Cation (M)</th>
<th>Anion (X)</th>
<th>a (Å)</th>
<th>b (Å)</th>
<th>c (Å)</th>
<th>α (°)</th>
<th>β (°)</th>
<th>γ (°)</th>
<th>Volume (Å(^3))</th>
<th>Channel size (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>O</td>
<td>6.985</td>
<td>6.990</td>
<td>10.649</td>
<td>90.86</td>
<td>89.70</td>
<td>89.88</td>
<td>520</td>
<td>1.43</td>
</tr>
<tr>
<td>Ge</td>
<td>O</td>
<td>7.151</td>
<td>6.976</td>
<td>10.709</td>
<td>90.47</td>
<td>89.66</td>
<td>89.81</td>
<td>534</td>
<td>1.46</td>
</tr>
<tr>
<td>Sn</td>
<td>O</td>
<td>7.499</td>
<td>6.821</td>
<td>10.966</td>
<td>89.70</td>
<td>90.12</td>
<td>89.35</td>
<td>561</td>
<td>1.50</td>
</tr>
<tr>
<td>Si</td>
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<td>8.848</td>
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<td>1.84</td>
</tr>
<tr>
<td>Ge</td>
<td>S</td>
<td>8.561</td>
<td>8.847</td>
<td>12.929</td>
<td>91.97</td>
<td>90.63</td>
<td>90.24</td>
<td>979</td>
<td>1.84</td>
</tr>
<tr>
<td>Sn</td>
<td>S</td>
<td>8.666</td>
<td>8.950</td>
<td>13.133</td>
<td>91.97</td>
<td>90.58</td>
<td>90.08</td>
<td>1018</td>
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<tr>
<td>Al</td>
<td>S</td>
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<td>8.567</td>
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<td>89.47</td>
<td>1021</td>
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<tr>
<td>P</td>
<td>S</td>
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<td>89.78</td>
<td>984</td>
<td>1.87</td>
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<tr>
<td>Si</td>
<td>Se</td>
<td>9.040</td>
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<td>91.89</td>
<td>90.74</td>
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<tr>
<td>Ge</td>
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<td>9.400</td>
<td>13.690</td>
<td>91.96</td>
<td>90.72</td>
<td>90.31</td>
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</tr>
<tr>
<td>Sn</td>
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<td>9.084</td>
<td>9.434</td>
<td>13.797</td>
<td>92.09</td>
<td>90.70</td>
<td>90.17</td>
<td>1181</td>
<td>1.97</td>
</tr>
</tbody>
</table>
while the selenides have channel sizes that are around 7% larger. We may also observe that the cation M has a relatively small effect on the size of the channels.

**Bandgaps**

To evaluate the intrinsic redox stability of the various LMPX compounds, we calculated the density of states (DOS) of all LMPX compounds using the HSE screened hybrid functional. Given that the calculated DOS of all LMPX compounds show similar trends regardless of the cation M, only the calculated DOS for Li_{10}GeP_{2}X_{12} for X = O, S and Se are shown in Fig. 2. We may make the observation that the O-substituted LMPO materials have a larger bandgap than the LMPS materials, which in turn have a larger bandgap than the Se-substituted LMPSe materials. Furthermore, we also find that both the valence band maximum and the conduction band minimum are dominated by anion states, regardless of the anion chemistry.

The bandgap of a material provides an upper limit on its electrochemical window.\(^{19}\) The DOS results predict that the O-substituted LMPO materials are expected to have greater intrinsic redox stability than the LMPS compounds, while the Se-substituted compounds are expected to have a lower intrinsic redox stability. This trend can be attributed to the increase in the energy levels of the valence p-orbitals of the anionic species as we move down the periodic table.

**Chemical stability with electrodes**

Chemical compatibility between the electrode and the electrolyte is important to prevent capacity degradation and impedance growth. In principle, the electrode material can react with the electrolyte through all of its components, though the reactivity with Li is most critical, as it is, by definition, a mobile species. On the anode side, the chemical potential of Li is very high, and the electrolyte can undergo reduction with Li uptake. The cathode, on the other hand, is strongly oxidizing and can break down the solid electrolyte by extraction of Li.

To investigate the chemical compatibility of LMPX electrolytes under extreme conditions of Li chemical potential, we constructed the lithium grand potential phase diagrams for the Li–M–P–X systems and evaluated the phase equilibria at two extremes for the lithium chemical potential (\(\mu_{Li}\)): the bulk Li metal chemical potential \(\mu_{0}^{Li}\) corresponding to a Li metal anode and \(\mu_{0}^{Li} - 5\) eV corresponding to a charged high-voltage (5 V) cathode.

On the anode, the predicted phase equilibria generally comprise Li_{2}X, Li_{3}P and a Li_{x}M_{y} alloy. We expect these to have relatively good Li^+ conductivity.\(^{32,33}\) Hence, while the solid electrolyte is not stable against metallic Li, it is possible to form a conductive solid electrolyte interphase (SEI). However, the presence of metallic products in the decomposition is worrisome and may lead to thickening of this interphase with time.

**Table 3** Phase equilibria for the Li_{10}MP_{2}X_{12} composition at cathode and anode \(\mu_{Li}\). The anode \(\mu_{Li}\) is set at the chemical potential of bulk lithium \(\mu_{0}^{Li}\), while the cathode \(\mu_{Li}\) is set at \(\mu_{0}^{Li} - 5\) eV, corresponding to a high voltage 5 V cathode.

<table>
<thead>
<tr>
<th>Cation (\text{(M)})</th>
<th>Anion (\text{(X)})</th>
<th>Equilibrium phases at cathode, (\mu_{Li} = \mu_{0}^{Li} - 5) eV</th>
<th>Equilibrium phases at anode, (\mu_{Li} = \mu_{0}^{Li})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge O</td>
<td>GeP_{2}O_{7}, O_{2}</td>
<td>Li_{15}Ge_{4}, Li_{2}O, Li_{3}P</td>
<td>Li_{3}Al_{2}, Li_{2}S, Li_{3}P</td>
</tr>
<tr>
<td>Si O</td>
<td>SiP_{2}O_{7}, O_{2}</td>
<td>Li_{3}Si_{2}, Li_{2}O, Li_{3}P</td>
<td>Li_{3}Si_{2}, Li_{2}O, Li_{3}P</td>
</tr>
<tr>
<td>Sn O</td>
<td>SnP_{2}O_{7}, O_{2}</td>
<td>Li_{3}Sn_{2}, Li_{2}O, Li_{3}P</td>
<td>Li_{3}Sn_{2}, Li_{2}O, Li_{3}P</td>
</tr>
<tr>
<td>Ge S</td>
<td>GeS_{2}, P_{2}S_{5}, S</td>
<td>Li_{15}Ge_{4}, Li_{2}S, Li_{3}P</td>
<td>Li_{17}Sn_{4}, Li_{2}S, Li_{3}P</td>
</tr>
<tr>
<td>Si S</td>
<td>SiS_{2}, P_{2}S_{5}, S</td>
<td>Li_{15}Si_{4}, Li_{2}S, Li_{3}P</td>
<td>Li_{17}Sn_{4}, Li_{2}P, Li_{3}S</td>
</tr>
<tr>
<td>Sn S</td>
<td>SnS_{2}, P_{2}S_{5}, S</td>
<td>Li_{17}Sn_{4}, Li_{2}S, Li_{3}P</td>
<td>Li_{17}Sn_{4}, Li_{2}P, Li_{3}S</td>
</tr>
<tr>
<td>Al S</td>
<td>AlS_{2}, P_{2}S_{5}, S</td>
<td>Li_{17}Al_{2}, Li_{2}S, Li_{3}P</td>
<td>Li_{17}Sn_{4}, Li_{2}P, Li_{3}S</td>
</tr>
<tr>
<td>P S</td>
<td>P_{2}S_{5}, S</td>
<td>Li_{2}S, Li_{3}P</td>
<td>Li_{2}S, Li_{3}P</td>
</tr>
<tr>
<td>Ge Se</td>
<td>Ge_{2}Se_{3}, P_{2}Se_{5}, Se</td>
<td>Li_{15}Ge_{2}, Li_{2}Se, Li_{3}P</td>
<td>Li_{15}Ge_{2}, Li_{2}Se, Li_{3}P</td>
</tr>
<tr>
<td>Si Se</td>
<td>Si_{2}Se_{3}, P_{2}Se_{5}, Se</td>
<td>Li_{15}Si_{2}, Li_{2}Se, Li_{3}P</td>
<td>Li_{15}Si_{2}, Li_{2}Se, Li_{3}P</td>
</tr>
<tr>
<td>Sn Se</td>
<td>Sn_{2}Se_{3}, P_{2}Se_{5}, Se</td>
<td>Li_{15}Sn_{2}, Li_{2}Se, Li_{3}P</td>
<td>Li_{15}Sn_{2}, Li_{2}Se, Li_{3}P</td>
</tr>
</tbody>
</table>
On the cathode, however, the phase equilibria are highly dependent on the anion type. The phase equilibria for the oxides at \( \mu_{Li}^0 = 5 \text{ eV} \) generally comprise \( M_xP_yO_z \) and \( O_2 \) gas. This decomposition is likely to lead to significant problems for the long term stability of the electrolyte. For S and Se, the cathodic phase equilibria comprise \( P_2S_5 \) and \( PSe_2 \) respectively, which may form potentially good glassy ionic conductors and may be passivating (Table 3).

Li\(^+\) diffusivity and conductivity

To elucidate the factors affecting Li\(^+\) diffusivity in the LGPS structural framework, we performed \textit{ab initio} molecular dynamics (AIMD) simulations of various substituted LMPX compounds. Both cation and anion substitutions were investigated. For cation substitutions, we investigated both isovalent as well as aliovalent substitutions with charge neutrality maintained via changes in the Li\(^+\) concentration.

Effect of cation substitutions and Li\(^+\) concentration

Fig. 3(a) shows the calculated diffusivities of the isovalent-substituted LMPS structures, where \( M = \text{Si or Sn} \). The data for LGPS are included as well for comparison. We may observe that in general, isovalent cation substitutions have a relatively small effect on diffusivity in this structure. The activation energy and Li\(^+\) conductivity at 300 K for all Li\(_{10}M_xP_2S_{12}\) materials are the same within the error of our simulations, as shown in Table 4.

To explore the effect of Li\(^+\) carrier concentration on the diffusivity, we also performed aliovalent cation substitutions, \( i.e., P^{5+} \) or \( Al^{3+} \) for \( Ge^{4+} \) to form \( Li_9P_3S_{12} \) or \( Li_{11}AlP_2S_{12} \) respectively. While it can be seen from Fig. 3(b) that aliovalent cation substitutions have a slightly larger effect on the diffusivity than isovalent substitution, the calculated data in Table 4 show that the activation barriers and Li\(^+\) conductivities of \( Li_9P_3S_{12} \) and \( Li_{11}AlP_2S_{12} \) are not significantly different from LGPS based on a t-statistic test, which suggests that the effect of carrier

Table 4  
\( Li^+ \) conductivity of cation-substituted compounds \( Li_{10}xMP_2S_{12} \) (\( M = \text{Si, Sn, P and Al} \)) and anion-substituted compounds \( Li_{10}GeP_2X_{12} \) (\( X = \text{O, S and Se} \)) at 300 K

<table>
<thead>
<tr>
<th>Compound</th>
<th>( E_a ) (eV)</th>
<th>Conductivity (mS cm(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Li_{10}GeP_2S_{12} )</td>
<td>0.21 ± 0.04</td>
<td>13</td>
</tr>
<tr>
<td>( Li_{10}SiP_2S_{12} )</td>
<td>0.20 ± 0.03</td>
<td>23</td>
</tr>
<tr>
<td>( Li_{10}SnP_2S_{12} )</td>
<td>0.24 ± 0.03</td>
<td>6</td>
</tr>
<tr>
<td>( Li_9P_3S_{12} )</td>
<td>0.26 ± 0.09</td>
<td>4</td>
</tr>
<tr>
<td>( Li_{11}AlP_2S_{12} )</td>
<td>0.18 ± 0.06</td>
<td>33</td>
</tr>
<tr>
<td>( Li_{10}GeP_2O_{12} )</td>
<td>0.36 ± 0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>( Li_{10}GeP_2Se_{12} )</td>
<td>0.19 ± 0.04</td>
<td>24</td>
</tr>
</tbody>
</table>
concentration and cation substitution is small around the Li⁺ concentration of LGPS. The slightly higher Li⁺ conductivity and lower activation barrier for Li₁₁AlP₂S₁₂ compared to LGPS may be partially accounted for by the larger channel size in this material (Table 2).

Effect of anion substitutions

The calculated diffusivity of Li₁₀GeP₂X₁₂ with anion S, O, or Se is shown in Fig. 4. Unlike the case of cation substitutions, we find that anion substitutions in general have a significant effect on Li⁺ diffusivity in the LGPX structure. Li⁺ diffusivity is significantly slower in Li₁₀GeP₂O₁₂ than in Li₁₀GeP₂S₁₂. The calculated activation energy of Li⁺ diffusion for Li₁₀GeP₂O₁₂ of 0.36 eV is also significantly higher than the 0.21 eV of LGPS. As a result, the Li⁺ conductivity at 300 K is only 0.03 mS cm⁻¹ for Li₁₀GeP₂O₁₂, which is three orders of magnitude lower than LGPS. This decrease in Li conductivity is consistent with the general observations that sulfides tend to be much better Li-ion conductors than oxides,

that anion substitutions in general have a significant effect on Li⁺ diffusivity in the LGPS structure. Li⁺ diffusivity does not improve significantly compared to LGPS. The activation energies for Li₁₀GeP₂Se₁₂ and LGPS are the same within the error bar (Table 4). This result suggests that there is a critical diffusion channel size, beyond which Li⁺ diffusivity does not improve significantly.

Effect of lattice parameter changes

To investigate the effect of lattice parameter changes on diffusivity in the LGPS structure, we performed AIMD simulations on isotropically scaled LGPS structures for six different scaling factors, ranging from −4% to +4%. Our simulation results (Fig. 5 and Table 5) show that changes in lattice parameters have a significant effect on the Li⁺ diffusivity. As the lattice parameters are decreased by 1%, 2%, or 4%, the activation energy increases to 0.28 eV, 0.47 eV and 0.59 eV, respectively. The Li⁺ conductivity at room temperature drops by an order of magnitude when the lattice parameters are decreased by 1%, and by more than six orders of magnitude when the lattice parameters are decreased by more than 2%.

On the other hand, when the lattice parameters are increased, we observe a relatively small improvement in the Li⁺ diffusivity. Again, this result suggests that the Li diffusion channels in LGPS are already at a somewhat “optimal” size, and further increases in lattice parameters, be it via substitution with a larger anion or by artificially increasing the lattice parameters, have a small effect on the Li⁺ diffusivity.

Discussion

Li₁₀GeP₂S₁₂ (LGPS) has an unusually high Li-ion conductivity, but the use of Ge as a rare and expensive element, and the sulfide anion chemistry make it an unlikely candidate for large-scale application in Li-ion batteries. We investigated the factors influencing stability and diffusivity in the Li₁₀GeP₂S₁₂ structure by performing cation and anion substitutions to obtain Li₁₀MP₂X₁₂ compounds. In general, we find that cation substitutions have relatively small effects on stability and diffusivity in this structure, while anion substitutions have a much greater effect.

Oxides are easier to handle than sulfides, but our results indicate that there is little hope for an oxide version of LGPS. We find that while all Li₁₀MP₂X₁₂ (M = Ge, Si, or Sn and X = O, S or Se) compounds are thermodynamically somewhat unstable, the
oxides are predicted to be highly unstable with $E_{\text{decomp}}$ in excess of 90 meV per atom. The high decomposition energies suggest that an oxide version of the LGPS structure is unlikely to be synthesizable. This may be due to the high stability of Li$_2$PO$_4$, which competes for stability with Li$_{10}$MP$_2$O$_{12}$ as can be seen from Table 1. Our results further indicate that there would be other problematic issues with Li$_{10}$MP$_2$O$_{12}$ electrolytes. We find that the oxygen-substituted Li$_{10}$GeP$_2$O$_{12}$ compound has much lower diffusivity (two orders of magnitude) than the LGPS compound, making it considerably less interesting than LGPS. This decrease in diffusivity is most likely due to the much smaller anion radius and polarizability of the oxide anion compared to the sulfide anion; similar observations have been made on the difference in conductivity between the LISICON and thio-LISICON conductors.\(^{35}\) Finally, our phase stability calculations predict that in contact with cathodes, Li$_{10}$MP$_2$O$_{12}$ compounds may undergo Li loss accompanied by the release of O$_2$ gas, which may prove highly problematic for the long term stability of the electrolyte.

Substituting Se for S only increases the conductivity by a small amount. This is consistent with our investigation of the effect of the lattice parameter on the diffusivity in the LGPS structure: while a decrease in lattice parameters results in significantly lower diffusivity and higher activation barriers, increases in lattice parameters result only in marginally higher diffusivity and marginally lower activation barriers. These observations suggest that the LGPS compound already has somewhat the “ideal” channel size for Li$^+$ diffusivity in this particular structural framework.

The good news is that cation substitutions of Ge seem to have a very little effect on the performance and stability of LGPS. Neither its Li$^+$ conductivity nor its anodic and cathodic stabilities are significantly affected. This result may have been expected, given that the interactions between the cations M and Li$^+$ ions are screened by the S$^{2-}$ anions surrounding the cations. Somewhat surprisingly, we find that alogenavelt cation substitutions, with corresponding changes in the Li$^+$ concentration, also have a relatively small effect on the Li$^+$ conductivity in the LGPS structure. This is unlike the significant changes in Li$^+$ conductivity (orders of magnitude) observed in other Li-ion conductors (e.g., lithium lanthanum titanates and NASICON-type materials) with changes in the Li$^+$ concentration.\(^{2,3}\) We believe that this is because the Li$_{10}$MP$_2$X$_{12}$ composition already has partial occupancies and most of the Li$^+$ ions are mobile in this structure. Given these data, it seems surprising that so far, no other versions of this compound, other than with Ge, have been reported in the literature.

Finally, a comparison between the DOS in Fig. 2 and the anodic and cathodic decomposition reactions in Table 3 indicates the dangers of assessing the electrochemical stability of potential electrolyte materials based on the intrinsic redox stability alone; chemical compatibility of the electrolyte with electrode materials is an equally important, if not more important, consideration. In terms of redox stability, the HSE-calculated DOS suggest that the oxides would have much better intrinsic redox stability than sulfides and selenides. But while sulfides and selenides form solid reaction products against the cathode and anode, our calculations predict the release of O$_2$ gas when Li$_{10}$MP$_2$O$_{12}$ is in contact with a high voltage cathode, which would be detrimental for stable battery operation.

Conclusions

In conclusion, our investigation of substituted Li$_{10-x}$MP$_2$X$_{12}$ compounds was motivated by the need to address two key limitations of the LGPS superionic conductor, namely the high cost of germanium, and the air and moisture sensitivity of a sulfide-based chemistry.

Our results show that the first limitation may be addressed by substituting relatively cheap silicon or tin for germanium in this structure. Isovalent cation substitutions have a small effect on the similar phase stability, electrochemical stability and diffusivity in these structures, with similar phase stability, electrochemical stability and Li$^+$ conductivity predicted for Li$_{12}$SiP$_2$S$_{12}$ and Li$_{12}$SnP$_2$S$_{12}$ as for LGPS. Alogenalent cation substitutions (M = Al or P) with corresponding changes in the Li$^+$ concentration also seem to have a small effect on the Li$^+$ conductivity.

However, the second limitation cannot be addressed by a simple substitution of oxide-based chemistry for sulfide-based chemistry. The oxygen-substituted Li$_{10}$MP$_2$O$_{12}$ compounds generally have much worse phase stabilities (with equilibrium decomposition energies $>$90 meV), better intrinsic electrochemical stabilities and much lower Li$^+$ conductivity than their sulfide counterparts. The selenium-substituted Li$_{10}$MP$_2$Se$_{12}$ compounds show the opposite trend from the oxide materials, but the magnitude of the increase in Li$^+$ conductivity is much smaller than the decrease in conductivity observed for the oxides. In addition, we also studied the effect of lattice parameter changes on Li$^+$ conductivity in this material and found the same asymmetry in behavior between increases and decreases in the lattice parameters, i.e., decreases in the lattice parameters lower the Li$^+$ conductivity significantly, while increases in the lattice parameters increase the Li$^+$ conductivity only marginally. Based on these results, we conclude that the size of the S$^{2-}$ is near the ideal size for Li$^+$ conduction in this structural framework.

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References