ENEE324, Home assignment 1. Date due February 14, 2026, 11:59pm EST.

Instructor: `Alexander Barg`

Please upload your work as a **single PDF file** to ELMS (under the "Assignments" tab)

- Submissions on paper or by email will not be accepted.
- Please do not submit your solutions as multiple separate files (pictures of individual pages). Such submissions are difficult to grade and will not be accepted.

- Justification of solutions is required.
- Each problem is worth 10 points.

Problem 1. A password consists of 2 symbols chosen from $\{!, @, \#, \S\}$, followed by 3 letters, followed by 2 digits.

(a) How many different passwords are possible if repetition is allowed everywhere?

(b) How many passwords are possible if no character (symbol, letter, or digit) may repeat?

(c) How many passwords are possible if letters may repeat, but digits and symbols may not?

(a) $4^2 \cdot 26^3 \cdot 10^2 = 28121600$

(b) $(4\cdot 3) \times (26\cdot 25\cdot 24) \times (10\cdot 9) = 16\,848000$

(c) $(4\cdot 3) \times 26^3 \times (10\cdot 9) = 18\,982080$

Problem 2. A box contains 5 red balls, 4 blue balls, and 3 green balls. Two balls are drawn sequentially without replacement.

(a) How many ordered outcomes are possible? (*ordered* means that the choice RB is different from BR)

(b) How many outcomes result in drawing two balls of different colors?

(c) What is the probability that at least one of the two balls drawn is green?

(a) An outcome is a pair of balls we are observing. Accounting for the ordering, there are 9 possibilities

RR, BB, GG RB RG BR BG GR GB

This is because it makes no difference which of the red balls appears in the pair RB, etc.

(b) 6 out of 9 yield different colors

(c) Let A be the event in question. There are $\binom{12}{2}$ choices of 2 balls in total. Of them $\binom{9}{2}$ include no green balls.

$$P(A) = 1 - P(A^c) = 1 - \frac{\binom{9}{2}}{\binom{12}{2}} = 1 - \frac{9 \cdot 8}{12 \cdot 11} = 1 - \frac{6}{11} = \frac{5}{11}$$

Problem 3. You distribute 10 identical tokens among 4 distinct boxes.

(a) How many possible distributions are there?

(b) How many distributions are there if each box receives at least two tokens?

(c) How many distributions are there if no box receives more than 5 tokens?

(a). With no restrictions, the number of distribitions is the "with replacement, order doesn't matter" case of the table in lec. 3 / discussion 2.

$$\binom{n+k-1}{k-1} = \binom{10+4-1}{4-1} = \binom{13}{3} = \frac{13\cdot 12\cdot 11}{6} = 26\cdot 11 = \underline{286}$$

(b) 8 tokens are fixed; it remains to distribute 2 between 4 boxes.

$$\binom{2+4-1}{4-1} = \binom{5}{3} = \underline{10}$$

(c) Count the number of distributions in which one box receives ≥ 6 tokens. This box receives anywhere between 6 ... 10 tokens; the remaining tokens are distributed between the other 3 boxes. The number of choices is

$$\underset{6}{\binom{4+3-1}{3-1}} + \underset{7}{\binom{3+3-1}{3-1}} + \underset{8}{\binom{2+3-1}{3-1}} + \underset{9}{\binom{4-1}{3-1}} + \underset{10}{\binom{3-1}{3-1}} = 35$$

Or, place 6 in the box and distribute 4 among the 4 boxes:

$$\binom{4+4-1}{4-1} = 35.$$ [Note: generally, $\sum_{m=k}^{t}\binom{m}{k} = \binom{t+1}{k+1}$; $\sum_{m=2}^{6}\binom{m}{2} = \binom{7}{3}$]

Since the first box can be chosen in 4 ways, we obtain

$$\binom{13}{3} - 4\cdot 35 = 286 - 140 = \underline{146}$$

Problem 4. A committee of 3 people is chosen from a group of 6 faculty members and 4 graduate students.

(a) How many committees are possible?

(b) How many committees contain at least one graduate student?

(c) What is the probability that a randomly chosen committee contains more faculty members than graduate students?

(a) # of ways to choose 3 out of 6 w/out replacement:

$$\binom{10}{3} = \frac{10\cdot 9\cdot 8}{6} = 5\cdot 3\cdot 8 = 120$$

(b) # of committees with no grad students $= \binom{6}{3} = 20$

The remaining count is our answer: $\underline{100}$

(c) Any such committee includes 2 or 3 faculty

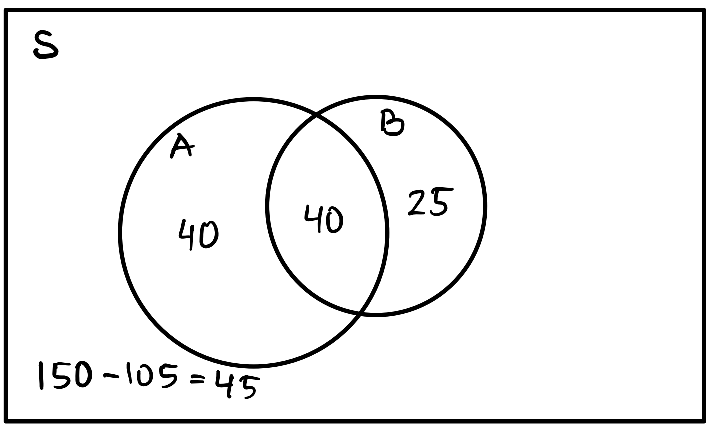
There are $4\binom{6}{2} + \binom{6}{3} = 60 + 20 = 80$ choices;

prob of such choice $= \frac{80}{120} = \frac{2}{3}$

Problem 5. At a conference with 150 attendees, 80 attended Workshop A, 65 attended Workshop B, and 40 attended both workshops.

(1) Draw a Venn diagram and label all regions with their numerical values.
(2) How many attendees attended neither workshop?
(3) How many attended exactly one workshop?
(4) How many attended Workshop B but not Workshop A?
(5) Express, in set notation and simplified form, the set of attendees who attended at most one of the two workshops.

(1)



(2) $S \setminus (A \cup B) = 45$

(3) $(A \cup B) \setminus (A \cap B) = 105 - 40 = 65$

(4) $B \setminus (A \cap B) = 25$

(5) $(A \cap B)^c = A^c \cup B^c$ Count $= 110$

Problem 6. For each comparison below, fill in the blank with $=$, $<$, or $>$, and briefly justify your answer.

(a) $\#$(ways to choose 3 books from 10) ____ $\#$(ways to choose 7 books from 10)

(b) $\#$(ways to arrange 5 people in a row) ____ $\#$(ways to seat 5 people around a circular table)

(c) P(rolling a sum of 7 with two dice) ____ P(rolling a sum of 8 with two dice)

(d) P(at least one 6 in two rolls) ____ P(exactly one 6 in two rolls) ?

(a) $\binom{10}{3} = \binom{10}{7}$ $\quad =$

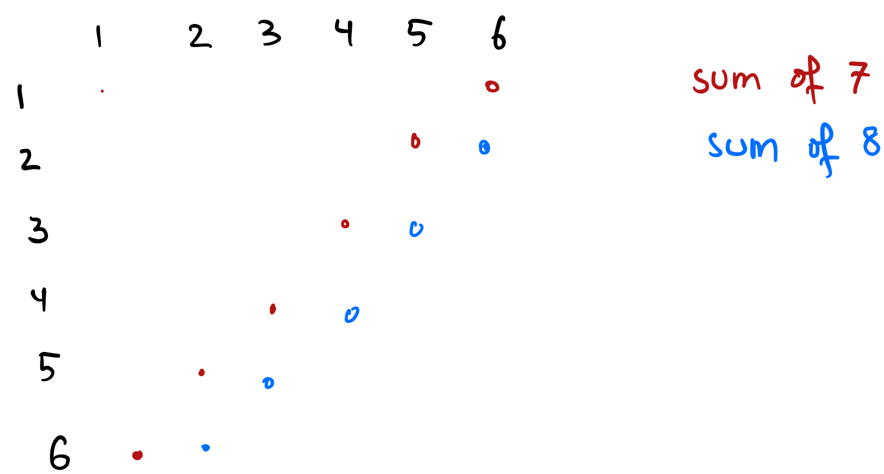
(b) $>$ since ABCDE and EABCD are different in the row but the same around a circular table.

The counts are $5! = 120$ and $\frac{1}{5}5! = 24$, respectively.

To see the last calculation, fix one person, say A, and permute the others in all the possible ways.

(c) sum of 8 is obtained in fewer ways since if 1^{st} die $= 1$, 8 cannot be obtained while 7 can. $>$

The counts are 6 and 5, prob.'s $= \frac{1}{6}$ and $\frac{5}{36}$



(d) $>$ since the first event includes everything in the second and the outcome $(6,6)$

The probabilities are $\frac{11}{36}$ and $\frac{5}{18}$, resp.