ENEE324, Home assignment 1. Date due September 13 2026, 11:59pm .

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Please upload your work as a **single PDF file** to ELMS (under the "Assignments" tab)

- Submissions on paper or by email will not be accepted.
- Please do not submit your solutions as multiple separate files (pictures of individual pages). Such submissions are difficult to grade and will not be accepted.
- Justification of solutions is required.
- Each problem is worth 10 points.

Problem 1. A hexadecimal device ID consists of 5 characters, each drawn from the 16 hex digits $\{0, 1, \ldots, 9, \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}$.

(a) How many device IDs are possible if repetition is allowed?

(b) How many device IDs are possible if no character may repeat?

(c) How many device IDs begin with a letter (one of A–F) and end with an even hex digit (one of $0, 2, 4, 6, 8, \mathrm{A}, \mathrm{C}, \mathrm{E}$), with the remaining three positions unrestricted (repetition allowed)?

Problem 2. A parts drawer contains 7 red, 5 blue, and 3 yellow resistors. Three resistors are drawn sequentially without replacement.

(a) How many ordered outcomes are possible? (*ordered* means the sequence R,B,Y is different from B,R,Y)

(b) How many ordered outcomes consist of three resistors of three different colors?

(c) What is the probability that at least one of the three resistors drawn is yellow?

Problem 3. You distribute 12 identical software licenses among 5 distinct departments.

(a) How many possible distributions are there?

(b) How many distributions are there if each department receives at least one license?

(c) How many distributions are there if no department receives more than 4 licenses?

Problem 4. A project team of 4 engineers is chosen from a pool of 7 hardware engineers and 5 software engineers.

(a) How many teams are possible?

(b) How many teams contain at least one software engineer?

(c) What is the probability that a randomly chosen team consists of exactly 2 hardware engineers and 2 software engineers?

Problem 5. In a class of 120 students, 70 know Python, 55 know C, and 45 know MATLAB. Moreover, 30 know both Python and C, 25 know both Python and MATLAB, 20 know both C and MATLAB, and 12 know all three languages.

(1) Draw a Venn diagram of the three sets and label all eight regions with their numerical values.
(2) How many students know none of the three languages?
(3) How many students know exactly one of the three languages?
(4) How many students know Python but neither C nor MATLAB?
(5) Express, in set notation, the set of students who know at least two of the three languages, and give its size.

Problem 6. For each comparison below, fill in the blank with $=$, $<$, or $>$, and briefly justify your answer.

(a) $\binom{9}{4}$ ____ $\binom{9}{5}$

(b) $\#$(distinct arrangements of the letters of "LEVEL") ____ $\#$(distinct arrangements of the letters of "LEVER")

(c) P(sum of 6 with two fair dice) ____ P(sum of 9 with two fair dice)

(d) P(at least one head in 3 fair flips) ____ P(exactly two heads in 3 fair flips)

Problem 7 (Monte Carlo simulation)**.** This problem asks you to *estimate* probabilities by simulation and compare them to the exact values. Use Python with NumPy (or an equivalent tool). Submit your code, your numerical estimates, and a short discussion.

(a) Write a simulation that reproduces the experiment of Problem 2: draw 3 resistors without replacement from a drawer of 7 red, 5 blue, and 3 yellow. Using $N = 10^6$ trials, estimate P(at least one yellow) and compare to the exact value $47/91$.

(b) *Birthday problem.* In a room of 25 people with birthdays independent and uniform over 365 days (ignore leap years), estimate P(at least three people share a birthday) using $N = 10^6$ trials, and compare to the exact value

$$1 - \frac{1}{365^{25}} \sum_{j=0}^{12} \binom{365}{j} \binom{365-j}{25-2j} \frac{25!}{2^j},$$

in which the term indexed by j counts the outcomes where exactly j birthdays are shared by two people each and the remaining $25 - 2j$ people have birthdays of their own.

(c) For part (b), run the simulation with $N = 10^3, 10^4, 10^5, 10^6$ and tabulate the estimate against N. Comment on how the error behaves as N grows.