ENEE324, Home assignment 4. Date due October 13, 2025, 11:59pm EDT.

Instructor: `Alexander Barg`

Please upload your work as a **single PDF file** to ELMS (under the "Assignments" tab)

• Submissions on paper or by email will not be accepted.
• Please do not submit your solutions as multiple separate files (pictures of individual pages). Such submissions are difficult to grade and will not be accepted.

• Justification of solutions is required.
• Each problem is worth 10 points unless noted otherwise.

Problem 1. Let X be a random variable that takes values $\{1, 2, 3, 4\}$ with probabilities $\{0.1, 0.3, 0.4, 0.2\}$. Define a random variable $Y = 5 - X$.

1. Find the PMF of Y.
2. Compute the expectation $E(X + Y)$.
3. Are X and Y independent? Justification required.

Problem 2. A factory produces microchips of 2 types, 70% of type A and 30% of type B. Chips of type A last a random number X_A of years with pmf $p_{X_A}(k) = \frac{1}{4}(\frac{3}{4})^{k-1}, k = 1, 2, \dots,$ and chips of type B last X_B years with pmf $p_{X_B}(k) = \frac{1}{2}(\frac{1}{2})^{k-1}, k = 1, 2, \dots.$

1. Verify that $p_{X_A}(k)$ and $p_{X_b}(k)$ are valid pmf's. Justification required.

A chip is chosen at random. Let X denote its random lifetime.

2. Find the pmf of X.
3. Compute the expectation $E(X)$.
4. Compute $P(X \geq 4)$.

Problem 3. There are 7 levels in a video game. You start at level 1 and have probability p_1 of advancing to level 2 (so with probability $1 - p_1$ your game ends at level 1, without ever advancing). If you reach level j, your probability of advancing to level $j + 1$ is p_j, for all $j = 1, 2, \dots, 6$. Denote by X the number of the highest level that you reach.

1. Find the pmf of X in terms of the numbers p_j.
2. Letting $p_j = 0.8 - 0.1j, j = 1, 2, \dots, 6$, find EX, the expected level to which you advance.

Problem 4. In a lottery, the probability for each ticket to be winning is p. You hope that buying 3 tickets will triple your chances of having at least one winning ticket.

1. What is the distribution of the number of winning tickets out of the three tickets you bought ? (find the pmf of the number X of winning tickets)

2. Find the probability that at least one of the three tickets is winning. Obtain the answer in two different ways:
2a. Use the inclusion-exclusion formula.
2b. Start with computing the probability of the complementary event.

3. By now you are convinced that your hope is not going to materialize. For which values of p your belief of tripling your chances is validated with no more than 1% margin of error? (hint: for small p).

Problem 5. (a) Let X be a random month of the year, numbered from 1 to 12, and let Y be the month after that, also represented as a number. Do X and Y have the same distribution? What is $P(X < Y)$?

(b) Suppose that two discrete RVs X and Y have the same distribution, but $P(X < Y) \geq p$. Is it possible that

- $p = 0.99$?
- $p = 0.999$?
- $p = 1$?

In each case give an example of random variables X, Y with $P(X < Y) \geq p$ or show that it is impossible.

Problem 6. In the home assignments for some imagined class there were different 100 problems. 10 problems out of this list, randomly selected, will appear on the exam. You pass if you solve correctly 7 problems out of the 10 included in the exam paper. You decide to boost your chances by memorizing solutions of s problems out of the entire set of 100, where $0 \leq s \leq 100$.

(a) Let X be the number of problems appearing in the midterm paper that you have memorized. What is the distribution of X? Identify it as one of the named distributions studied in class and give the pmf $p_X(k)$ in terms of s.

(b) Find the smallest number s such that your chance of success is at least 50%.