ENEE324-03. Problem set 1

Date due February 10, 2015

1. Use Venn diagrams to show that

$$P((A \cup B) \cap C) = P((A \backslash B) \cap C) + P((B \backslash A) \cap C) + P((A \cap B) \cap C)$$

(the notation $A \backslash B$ refers to the set $\{x \in A\} \cap \{x \notin B\}$).

2. Suppose our experiment is rolling two 6-sided dice. Let Y be the minimum of the two rolls. Find the probability that $Y = i$ for all possible values of i.

3. Consider rolling a 6-sided die. Let A be the set of outcomes where the roll is an odd number and let B be the set of all outcomes where the roll is 4 or more. Compute the events $E_1 = (A \cap B)^c$ and $E_2 = (A \cup B^c)^c \cup (A \cup B)^c \cup (A^c \cup B)^c$ (list all the possible outcomes that constitute the event E_1 and all the possible outcomes of the event E_2). Draw a Venn diagram showing events E_1 and E_2.

4. Let $A, B \subset \Omega$, $P(A) = 0.5, P(B) = 0.6$. What are the maximum and minimum possible values for $P(A \cap B)$?

5. Two players take turns at tossing a fair coin. The first of them to get Heads wins. What is the probability that Player 1 wins? (Answer: 2/3).

6. Events A and B are called independent if $P(A \cap B) = P(A)P(B)$. Two real numbers $x, y \in [0, 1]$ are selected at random according to a uniform probability distribution. Let

$$A = \{x \geq 1/3\}, \quad B = \{y \leq 1/3\}, \quad C = \{x \geq y\}.$$

Are A and B independent? Are A and C independent?