

ENEE324-03. Problem set 1

Date due September 17, 2014

1. Use Venn diagrams to show that

$$\begin{aligned}\mathbf{P}(A \cup B \cup C) &= \mathbf{P}(A) + \mathbf{P}(B) + \mathbf{P}(C) \\ &\quad - \mathbf{P}(A \cap B) - \mathbf{P}(B \cap C) - \mathbf{P}(A \cap C) + \mathbf{P}(A \cap B \cap C).\end{aligned}$$

2. Six balls $b_1, b_2, b_3, b_4, b_5, b_6$ are placed in 3 urns randomly as follows: three balls are placed in the first urn, two of the remaining three balls are placed in the second urn, and the last ball is placed in the third urn. What is the size of the sample space of this experiment (how many different outcomes does it contain)?

3. Four classmates have their birthdays in May. Assuming that all the dates in May are equally likely, compute the probability that their birthdays all fall on different dates.

4. Consider an experiment in which a fair coin is tossed 4 times.

Let A be the event that the outcome contains 2 or more heads.

Let B be the event that the outcome begins with TT.

Let C be the event that the outcome contains an odd number of tails.

Two events $A, B \subset \Omega$ are called *independent* if $P(A \cap B) = P(A)P(B)$.

- (a) Determine whether A and B are independent.
- (b) Determine whether A and C are independent.
- (c) Determine whether B and C are independent.
- (d) Determine whether A, B and C are independent.
- (e) Determine whether A and B are independent given that C occurred.

5. Two real numbers $x, y \in [0, 1]$ are selected at random according to a uniform probability distribution. Let

$$A = \{x \geq 1/3\}, \quad B = \{y \leq 1/3\}, \quad C = \{x \geq y\}.$$

Are A and B independent? Are A and C independent?

6. There are 3 hard-cover and 4 paperback books on a shelf. 3 books are removed one by one, without replacement.

Draw a fully labelled tree diagram of the experiment.

What is the probability that the outcome contains 2 papersbacks and one hard-bound book?