

## ENEE324 Fall 2016. Problem set 2

Date due September 14, 2016

### Please justify your answers

1. Consider an infinite series of flips of a fair coin  $F_1, F_2, \dots$ . Identify the sample space  $\Omega$  of the experiment. What is the cardinality of  $\Omega$ ? What is the probability that
  - (a) there is at least one  $H$  among  $F_1, \dots, F_n$  (the first  $n$  flips)?
  - (b) there are exactly  $k$  outcomes  $H$  among the first  $n$  flips?
  - (c) In the sequence  $F_1, F_2, \dots$  the pattern  $HHH$  appears for the first time before the first appearance of the pattern  $THH$ ?
  - (d) all of the outcomes  $F_1, F_2, \dots$  are  $H$ ?
2. (tree diagrams) Network switches are produced either by Factory 1 (40%) or Factory 2 (60%). The proportion of switches that is faulty is 3% for Factory 1 and 5% for Factory 2. You have purchased 5 network switches; what is the probability that one of them is faulty?
3.  $n$  cards are labelled  $1, 2, \dots, n$ . We select 2 cards out of them randomly without replacement. What is the probability that the first card has smaller value than the second one?
4. Urn 1 contains 6 blue and 5 green marbles, and urn 2 contains 9 blue marbles. One of the urns is selected at random, then three marbles are drawn, and all three are blue. What is the probability that Urn 1 was selected?
5. (Total probability formula). A roll of two dice produces two numbers. We roll two (fair, 6-sided) dice six times. What is the probability that the sum of the numbers in roll 6 has not occurred in any of the previous five rolls?
6. Two events  $A$  and  $B$  are called independent if  $P(AB) = P(A)P(B)$ . We select a random point in the interval  $[0, 1]$  of the real axis. Let  $A = [1/4, 3/4]$  (i.e., the event that the point chosen is somewhere between  $1/4$  and  $3/4$ ), let  $B = [1/2, 1]$ , and let  $C = [1/4, 1/2] \cup [3/4, 1]$ . Are the following events independent:
  - (a)  $A$  and  $B$ ?, (b)  $A$  and  $C$ ?, (c)  $A$  and  $BC$ ?, (d)  $A$  and  $(B \cup C)$ ?