ENEE324-03 Spring 2019. Problem set 3

Date due February 26, 2019

Please justify your answers

Problem 1: Out of 9 mathematicians and 12 engineers, a committee consisting of 4 mathematicians and 5 engineers is to be formed. In how many ways can this be done if

(a) any mathematician and any engineer can be included?

(b) one particular engineer must be in the committee?

(c) two particular mathematicians cannot be in the committee?

Problem 2: There are n vehicles of which m are passenger cars and $n-m$ trucks, lined in a single lane in front of a traffic light. Your task is to find the number of different orderings of the vehicles. The passenger cars are indistinguishable, which means that car 1 before car 2 or car 2 before car 1 (possibly with some other vehicles separating them) count for just one configuration. The trucks are also indistinguishable. (compare with problem 23, p. 116 in the Textbook)

Problem 3: Show that $\binom{n}{k} = \sum_{i=k}^{n} \binom{i-1}{k-1}, n \geq k$. (Hint: consider k-subsets of the set of numbers $\{1, \ldots, n\}$. How many subsets have i as their largest entry?)

Problem 4: We are testing a statistical hypothesis about an experiment that can have 3 possible outcomes. Suppose that α_i is the probability of not detecting outcome $i, i = 1, 2, 3$ when this outcome has in fact occured. Find the conditional probabilities p_i of each of the possible outcomes $i = 1, 2, 3$ conditional on the fact that our test *has not* detected outcome 1.

Problem 5: I buy a package of cereal every week. Each package contains a coupon for a company's product. There are n types of coupons in total, and coupon i appears with probability $p_i, i = 1, \ldots, n$ independently of everything else. After k weeks I have as many coupons. Let A_i be the event that I have in my possession at least one coupon of type i.

For $i \neq j$ find (a) $P(A_i)$, (b) $P(A_i \cup A_j)$, (c) $P(A_i|A_j)$.

Problem 6: A person contracts a particular (not too serious) disease at a Poisson rate $\lambda = 4$/yr. A vaccination, given to a large population group, is known to reduce the rate to $\lambda = 3$ in 60% cases and to have no effect in the remaining 40% of them. My friend has submitted himself to vaccination for 2 years and has had 3 onsets of the disease during that time. What is the probability that the drug has a positive effect for him?

Problem 7: A DC-bound metro train arrives at the College Park station every 10 minutes starting at the whole hour. A passenger enters the station at a time uniformly distributed between 9:10 and 9:30. What is the probability that he waits (a) less than 5 minutes, (b) more than 7 minutes?