ENEE324. Problem set 3 [1]

Issued on September 29, **Date due** October 8, 2014

3.1. A Poisson r.v. X has the mean value $\mathbf{E}[X] = 3$. What is the probability $\mathbf{P}(X \le 3)$? $\mathbf{P}(X > 0)$?

3.2. The lab counter is programmed to record one elementary particle per second. An electron appears with probability 0.2, a proton with probability 0.3, a neutrino with probability 0.4, no particles are recorded with probability 0.1. What is the probability that

(a) the first electron appears in the 4th observation?
(b) the first proton appears no earlier than the 5th observation?
(c) in the first 10 seconds there are exactly 3 electrons, 2 protons, and 1 neutrino?

3.3. Let X and Y be geometric r.v.'s with $p_X(k) = p_Y(k) = p(1-p)^{k-1}, k = 1, 2, \ldots,$ where $0 < p < 1$.

(a) Compute the PMF $\mathbf{P}(X + Y = 10)$.

(b) Compute the conditional PMF $\mathbf{P}(X = k | X + Y = 10)$.

3.4. Let X_1 be a Bernoulli r.v. with $p_X(1) = p_1$ and X_2 be a Bernoulli r.v. with $p_X(1) = p_2$. Write out the p.m.f. for $Z = X_1 - X_2$ and compute $\mathbf{E}(Z), \text{var}(Z)$.

3.5. Let X and Y be independent r.v.'s with

$$p_X(k) = \begin{cases} 0.2 & k = 0 \\ 0.3 & k = 1 \\ 0.4 & k = 2 \\ 0.1 & k = 3 \end{cases} \qquad p_Y(k) = \begin{cases} 0.7 & k = 1 \\ 0.2 & k = 3 \\ 0.1 & k = 4 \end{cases}$$

Write out the p.m.f. of the r.v. $Z = \min(X, Y)$.

3.6. Let X and Y be two Bernoulli random variables taking values 0 and 1. The joint p.m.f. p_{XY} is given by

$$p_{XY}(x, y) = \begin{cases} \frac{1}{2}(1-p) & x = y \\ \frac{1}{2}p & x \ne y. \end{cases}$$

Compute $p_Y(0), p_Y(1), p_{Y|X}(0|1), p_{Y|X}(0|0)$.

[1] r.v. stands for "random variable," p.m.f. stands for "probability mass function."