ENEE324-03. Problem set 4

Date due March 10, 2015

1. (a) Take a well-shuffled deck of 52 playing cards and start opening cards one by one from the top. How long is the expected wait till the first ace?

(b) Now instead of opening cards from the top, draw a card randomly, and if not an ace, put it back in the deck. What is the probability that you will use at least 10 draws before seeing an ace?

2. X is a discrete RV with $EX = 1$ and $E[X(X-2)] = 3$. Find $\text{Var}(-3X+5)$.

3. A tipsy guest is leaving a party, and is faced with the choice of n coats. He picks them up randomly and tries on, if it's not his, he returns it to the coat hanger, whereupon he again faces a choice of n coats. What is the expectation and the variance of the number of tries till he finally departs? The same questions if, after realizing that the coat is not his, he instead tosses it on the floor.

4. An undergraduate program in some university claims that only 3% of students do not graduate after 5 years. A random selection of 24 students were observed till they graduated, and it was found that two took more than 5 years. Is it fair to reject the claim of the administration based on this observation?

5. Let X and Y be two Bernoulli RVs taking values 0 and 1. The joint pmf p_{XY} is given by

$$p_{XY}(i,j) = \begin{cases} \frac{1}{2}(1-p), & i = j \\ \frac{1}{2}p & i \neq j \end{cases}$$

Compute $p_Y(0), p_Y(1), p_{Y|X}(0|1), p_{Y|X}(0|0)$.

6. Let X and Y be geometric RVs with $p_X(k) = p_Y(k) = p(1-p)^{k-1}, k = 1, 2, \ldots,$ where $0 < p < 1$. Compute the probability $P(\{X + Y = 10\})$. Compute the conditional pmf $P(X = i | X + Y = 10), i = 1, 2, \ldots.$