ENEE324 Fall 2016. Problem set 4

Date due October 3, 2016

Please justify your answers

1. Let X_1 be the random outcome of a roll of a fair 6-sided die, and let X_2 be the outcome of the second (independent) roll of the same die. Define the following RVs:

$$Y_1 = \max(X_1, X_2),\ Y_2 = \min(X_1, X_2),\ Y_3 = X_1 + X_2,\ Y_4 = X_1 - X_2$$

Determine the range and the PMF of each of the variables $Y_i, i = 1, 2, 3, 4$. (Hint: In each case, group the values of X_1 and X_2 that give the same value of Y_i and use the formula for the PMF of the function of an RV.)

Compute the expectation and the variance of Y_1, Y_2.

2. From a parking lot with 15 sedan cars and 20 SUVs we select a random group of 10 vehicles. Let X be the number of SUVs in the group. Find the PMF of X.

3. There's an urn with 6 balls, each of which is labelled with a number from the set $\{1, 5, 10\}$. There is one ball labelled 10, two labelled 5, and three labelled 1 in the urn. You draw two balls at random and win the amount of money equal to the sum of their labels. What is the minimum entry fee to play the game so that on average the house earns a profit? (Hint: Consider an RV X equal to your net pay, which is the win minus the entry fee).

4. Define an RV X equal to the number of tosses of a fair coin until for the first time the sequence of outcomes contains at least one H and at least one T. Compute EX. (Your answer should be a single number, not a sum or any other expression.)

5. You are standing on the real line at the origin. Each second you jump left 1 foot with probability 1/3 or jump right 1 foot with probability 2/3. The jumps are independent of each other. What is the probability that after 6 seconds you find yourself standing at the origin?

6. There is a Poisson RV X such that $p_X(1) = p_X(4)$. Find EX and $\mathrm{Var}(X)$. (Your answer for each of these should be a single number, not a sum or any other expression.)