ENEE324. Problem set 5

Date due March 30, 2016

Explanations are required, no credit for just the answer.

1. Consider the function $\phi(x) = 0, x < 1$ or $x > 5$ and

$$\phi(x) = \frac{1}{2} - \frac{1}{4}|x - 3|, \quad 1 \le x \le 5.$$

Plot the graph of $\phi(x)$. Does it represent a valid pdf? If yes, compute the cdf (make sure you give the answer for all $x, -\infty < x < \infty$.

2. Let $f_X(x) = 2x\pi^{-2}, 0 < x < \pi$. Define the RVs $Y = \sin(X), Z = \sin(X/2)$. Find $f_Y(y), f_Z(z)$ (you can use the method described on p.202 of the textbook).

3. A snowstorm continues for 1 hour, and the accumulation of snow on the ground in a particular spot it described by an RV X with pmf $f_X(x) = 6x(1 - x), 0 < x < 1$ and $f_X(x) = 0$ otherwise. What's the probability that the amount of snow at the end of the storm is within two standard deviations of the mean?

4. Packets arrive at a server one by one with exponentially distributed time between arrivals, $f_T(t) = \lambda e^{-\lambda t}, t \ge 0$. (a) What is the probability that there are no arrivals in the next t minutes? (b) What is the probability that the next packet arrives within t minutes, $a \le t \le b$, where a, b are some given numbers?

5. We choose a random point X in the segment $[0, \pi/4]$. Find $E(\cos 2X), E(cos^2(X))$.

6. A random point X is chosen from the interval $(0, 1)$. What is the distribution of the RV $Y = 5X - 1$?