ENEE324. Problem set 5

Date due: Nov. 12, 2014

1. The reading of my car's speedometer is on average accurate, but has a random error with average 0.25 mph, neglecting sign. What is the probability that at a given moment, the speed measured exceeds the actual speed by more that the average error? Assume the Gaussian model for the errors.

2. A mile relay team consists of four runners. The time each takes to run its part of the race is $\mathcal{N}(52, (0.8)^2)$ with time in seconds (these r.v.'s are pairwise independent). Find the probability that the team will complete the race in less than 3 minutes 26 seconds. Please provide a numerical answer.

3. Let X and Y have the joint pdf

$$f_{XY}(x,y) = \begin{cases} ce^{-x-y} & 0 \le y \le x < \infty, \\ 0 & \text{for all other pairs } x, y \end{cases}$$

Find c, $f_X(x), f_Y(x), F_{X,Y}(x,y), \mathbf{E}(X^2|Y=2), \mathbf{P}[X+Y \le 1]$.

4. Let $X \sim \text{Unif}\,[0,1]$ be a random variable and let

$$Y = \ln \frac{X}{1-X}.$$

Find the CDF $F_Y(y)$ and the PDF $f_Y(y)$. Make a sketch of $f_Y(y)$.

5. Let $X \sim \text{Unif}\,[0,1], Y \sim \exp(\lambda)$ be independent r.v.'s. Let $Z = X + Y$.

(a) Find the PDF $f_Z(z)$ and the CDF $F_Z(z)$. Make a plot of $f_Z(z)$ for $\lambda = 1/2$.

(b) Find $M_Z(s)$. For which s is it defined?

(c) Find $\mathbf{E}(Z^2)$.

6. Let $X \sim \text{Lalpace}(2)$. Let $Z = X + Y$ where $Y \sim \text{Unif}\,[0,2]$.

(a) Compute $\mathbf{P}(Z > 2|Y|)$.

(b) Find the PDF $f_Z(z)$.

7. A point D is chosen at random from a triangle with sides $3, 4, 5$. Find the probability that the side of the triangle closest to D is the hypotenuse.

8. Two random variables, X and Y, are uniformly distributed in the triangle $\{x = 0, y = 0, x + y = 2\}$. Are they positively or negatively correlated or uncorrelated?