ENEE324. Problem set 6

Date due April 6, 2016

Explanations are required, no credit for just the answer.

1. (a) Let $X \sim \mathcal{N}(0, \sigma^2)$. Let $Y = X^2$. Find the pdf of Y.

(b) Let $X \sim \mathcal{N}(2, 3), Y = X^2$. Find $P(X > 1), P(Y > 1)$.

2. We are given RVs X and Y.

(a) Suppose that

$$f_{XY}(x, y) = \begin{cases} 2x & \text{if } 0 \le x, y \le 1 \\ 0 & \text{o/w} \end{cases}$$

Find the cdf and pdf of $Z = X - Y$.

(b) Suppose that $f_{XY}(x, y) = e^{-(x+y)}, 0 \le x < \infty; 0 \le y < \infty$ and $f_{XY}(x, y) = 0$ if $xy < 0$. Find the pdf of $Z = X/Y$.

3. The interval between 2 arrivals of a query to the server is an RV X with PDF $f_X(x) = \lambda x e^{-x}, x > 0$ and 0 o/w, measured in seconds. (a) Find λ. (b) Find the CDF of X. (c) What is the probability that, if a query arrived just now, the next one will arrive in 3 to 6 minutes? In at least 8 minutes?

4. We are given two RVs X and Y with joint PDF $f_{XY}(x, y) = ce^{-x}$ if $x \ge 0, |y| < x$ and 0 o/w. (a) Find the value of c. (b) Find $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$. (c) Find $E(Y|X = x)$ and $\mathrm{Var}(Y|X = x)$.

5. We are given two independent exponential RVs X and Y with the parameters λ_1 and λ_2, respectively. Suppose that they model arrivals of two different types of queries to the server. Let Z be the random time, counting from 0, of the arrival of the first query to the server (it doesn't matter if it's type one or type-two query). Find the CDF $F_Z(z)$.